MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. J. W. L. GLAISHER, M.A., F.R.S., PRESIDENT, in the Chair.

Henry Bourget, D.-ès-Sc., Astronome-adjoint à l'Observatoire de Toulouse, 20 Rue Saint Jacques, Toulouse ;
Major John Cassells, J.P., 154 Queen's Drive, Crosshill, Glasgow ;
Patrick Sinclair Hardie, M.A., B.Sc., 305 Onslow Drive, Dennistoun, Glasgow ; and
Richard Kerr, F.G.S., 13 Ormiston Road, Greenwich, S.E.,

were balloted for and duly elected Fellows of the Society.

Seventy-eight presents were announced as having been received since the last meeting, including amongst others :—

Two drawings of the total Solar Eclipse of 1901 May 18, made from negatives taken at Mauritius with the Newbegin telescope, presented by Mrs. Maunder ; lantern slide from photograph of a Leonid Meteor, taken 1901 November 14, at the Goodsell Observatory, Carleton College, Minn., presented by W. W. Payne ; N. M. Kam, Catalog von Sternen deren Oerter durch selbständige Meridian Beobachtungen bestimmt worden sind, herausgegeben von H. G. van de Sande Bakhuyzen, presented by the Amsterdam Academy.

Preliminary Note on the Possible Existence of Two Independent Stellar Systems. By F. A. Bellamy and H. H. Turner.

The publication of the above paper is deferred, as the authors find some numerical corrections necessary.

Statistics of Stars in a Zone of 5° from +65° to +70° Decl. counted on Photographs for the Astrographic Chart and Catalogue at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

An important use to which the long-exposure photographs for the astrographic chart can be immediately put is that of star gauging. In combination with counts of the number of stars shown with shorter exposures, these photographs give down to their limit information of a very interesting and complete character as to the distribution of the stars.

The following paper gives the results of these counts for the zone of 5° in width between +65° and +70° Decl.

The photographs were taken with the astrographic equatorial of 33 centimetres (or 13-inch) aperture and $3^m{\cdot}43$ (or 11-foot 3-inch) focus, and were usually obtained when the field photographed was within one hour of the meridian at upper culmination, and never more than one hour and a half from it. The most sensitive plates available were used—generally Ilford Special Rapid or Barnet Rocket plates. The chart plates have an exposure of 40^m, the catalogue plates three exposures of 6^m, 3^m, and 20^s respectively. Plates on any night are not considered satisfactory, and are rejected unless images of $9^m{\cdot}0$ stars appear with the exposure of 20^s on the catalogue plate.

In the counting of the chart plates with 40^m exposure the same plan is followed as in the measurement of the catalogue plates with the duplex micrometer; that is, the same field of the sky (usually of one square degree) is simultaneously examined on two plates. For example, the count of stars in the part of the sky lying between 65° and 66° of declination and between $0^h\ 0^m$ and $0^h\ 9^m$ of right ascension is made at the same time for the two plates whose centres are R.A. $0^h\ 9^m$ Decl. 65°, and R.A. $0^h\ 0^m$ Decl. 66°. The number of stars seen on both plates is counted, as well as the additional stars seen (with certainty) on one plate only. The counts are all made in duplicate (by two observers independently).

The counts of the stars shown on the catalogue plates are taken from the printed, but as yet unpublished, volume of the measures for the Greenwich Astrographic Catalogue Decl. +64° to +72°. In this volume the stars which are shown with an exposure of 20^s are indicated, as well as those stars which are shown with exposures of 3^m and 6^m. In the following table no discrimination has been made between the 6^m and 3^m exposures, but it is necessary to state that images of stars with 6^m exposure (which were not shown with 3^m) have not been measured unless

the star was shown on both plates. The possible combinations are :—

(i.) 6^m and 3^m images shown on both plates.
(ii.) 6^m and 3^m images shown on one plate, but only 6^m image on the other.
(iii.) 6^m and 3^m images on one plate, but the star not shown on the other.
(iv.) 6^m image, but not 3^m image shown on both plates.
(v.) 6^m image shown on one plate but not on the other.

In the fifth of these cases the image has not been measured, and consequently these cases are not included in the counts.

The following table gives the number of stars for areas of one degree in declination and 45^m in right ascension, these numbers being usually the sum of five different areas counted on six different plates, three in one zone and three in the zone next to it. This summation tends to smooth the inequalities due to the varying conditions of the nights when the plates were taken. The area included in 45^m of right ascension in the different zones of 1° in declination is :—

4·67	square	degrees	in zone	65°
4·48	,,	,,	,,	66°
4·31	,,	,,	,,	67°
4·12	,,	,,	,,	68°
3·94	,,	,,	,,	69°

The total number of stars found in the Bonn *Durchmusterung* for these areas and of the number of stars of the ninth magnitude, or brighter, are also included in the following table.

The table gives in parallel columns the stars shown in duplicate—*i.e.* on each of the overlapping plates—and the total number of stars shown with the various exposures.

TABLE I.

Number of Stars shown with Various Exposures for Areas of 1° *in Decl. and* 45^m *in R.A. in the Zones from* 65° *to* 70° *Decl.*

Limits of R.A.	Zone.	No. in B.D.		Number shown on Photographs.					
				Exposure 20s.		Exposures 3^m and 6^m.		Exposure 40^m.	
		$9^m{\cdot}0$ and brighter.	Total No.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.
h m	°								
0 0 to 0 45	+65	44	108	53	95	181	264	1071	1390
	66	26	75	51	73	237	253	890	1138
	67	29	73	47	68	265	295	1336	1496
	68	17	60	50	78	282	308	1246	1394
	+69	17	48	40	90	302	373	1513	1680

Limits of R.A.	Zone.	No. in B.D.		Number shown on Photographs. Exposure 20s.		Exposures 3m and 6m.		Exposure 40m.	
		9m·0 and brighter.	Total No.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.
h m	°								
0 45 to 1 30	+65	35	100	69	140	258	347	1073	1229
	66	20	65	36	81	199	261	702	1148
	67	21	66	25	39	140	165	1391	1587
	68	31	71	47	71	239	306	2122	2324
	+69	23	61	55	122	332	413	2792	2867
1 30 to 2 15	+65	29	70	25	71	173	219	985	1090
	66	22	69	36	61	176	218	837	1092
	67	20	65	39	56	197	241	958	1038
	68	16	49	43	59	277	317	1391	1475
	+69	13	33	36	68	255	314	1799	1923
2 15 to 3 0	+65	20	64	21	53	136	176	976	1270
	66	12	37	13	19	62	68	720	842
	67	21	45	25	41	112	157	768	909
	68	27	54	57	96	198	323	887	1100
	+69	21	54	68	130	361	420	1262	1410
3 0 to 3 45	+65	23	46	36	75	147	193	527	633
	66	21	53	29	49	157	238	771	1041
	67	18	49	31	51	188	307	1107	1251
	68	18	78	45	85	250	305	1355	1914
	+69	14	32	29	46	187	215	1287	1719
3 45 to 4 30	+65	17	47	34	55	144	180	975	1222
	66	14	64	30	33	150	166	1149	1307
	67	19	45	29	52	158	249	1204	1242
	68	18	51	26	58	147	200	1344	1488
	+69	16	38	21	46	160	228	1127	1410
4 30 to 5 15	+65	16	47	47	67	230	303	2306	2783
	66	21	50	28	48	210	271	2530	2945
	67	23	45	22	41	181	222	1964	2461
	68	20	46	31	66	278	385	1620	1786
	+69	29	51	56	83	361	425	1608	2234
5 15 to 6 0	+65	19	46	48	80	270	338	2162	2871
	66	14	37	26	63	201	340	1513	2306
	67	11	30	26	46	181	260	1441	1906
	68	13	39	43	72	321	448	1717	2068
	+69	15	47	41	65	292	375	1748	2030

Limits of R.A.	Zone.	No. in B.D.		Number shown on Photographs. Exposure 20s.		Exposures 3m and 6m.		Exposure 40m.	
		9m·0 and brighter.	Total No.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.
h m	°								
6 0 to 6 45	+65	18	33	48	63	254	299	1364	2164
	66	16	44	48	77	324	372	1172	1925
	67	13	45	33	57	250	355	1642	2115
	68	13	31	35	45	233	260	1737	1960
	+69	9	31	33	53	275	332	1655	1742
6 45 to 7 30	+65	8	34	25	43	172	206	1465	1929
	66	21	43	37	60	201	272	1166	1349
	67	21	41	32	61	250	411	1247	1523
	68	17	35	24	38	206	282	1452	1537
	+69	15	40	25	44	198	232	1235	1435
7 30 to 8 15	+65	17	45	36	61	204	257	1155	1395
	66	17	32	30	58	219	258	1153	1467
	67	14	37	30	52	243	301	1050	1425
	68	12	39	32	57	196	259	976	1253
	+69	17	34	20	37	169	206	1176	1370
8 15 to 9 0	+65	24	60	48	87	219	271	717	799
	66	15	53	40	68	214	263	912	1048
	67	13	28	25	48	168	233	873	1005
	68	13	26	22	38	167	184	891	984
	+69	10	43	38	59	206	271	989	1028
9 0 to 9 45	+65	16	54	21	36	130	157	773	901
	66	16	38	27	43	113	145	734	936
	67	23	44	38	49	145	164	747	951
	68	5	26	23	35	155	169	736	790
	+69	16	38	28	47	170	207	693	813
9 45 to 10 30	+65	17	47	37	52	160	192	749	868
	66	13	39	26	39	166	180	850	986
	67	17	40	29	40	150	170	750	921
	68	12	32	19	33	138	170	725	819
	+69	14	31	26	57	144	205	828	970
10 30 to 11 15	+65	17	42	45	65	202	248	684	771
	66	22	43	48	65	196	235	868	932
	67	11	26	27	38	135	169	830	888
	68	9	28	31	37	152	172	802	836
	+69	6	22	20	45	173	215	860	928

Limits of R.A.	Zone.	No. in B.D.		Number shown on Photographs. Exposure 20s.		Exposures 3m and 6m.		Exposure 40m.	
		9m·0 and brighter.	Total No.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.
h m	°								
11 15 to 12 0	+65	13	38	25	41	131	151	757	885
	66	12	30	19	43	153	250	805	889
	67	20	34	37	58	209	249	738	764
	68	16	33	39	51	215	249	724	748
	+69	19	37	47	89	249	346	812	945
12 0 to 12 45	+65	21	40	28	44	128	173	682	760
	66	11	24	17	24	101	136	733	831
	67	9	35	22	38	137	187	686	797
	68	7	20	22	41	163	198	688	739
	+69	18	38	40	73	217	291	816	879
12 45 to 13 30	+65	17	38	33	57	156	191	801	953
	66	11	34	30	46	146	170	752	821
	67	11	24	19	29	137	165	754	818
	68	16	34	29	48	189	219	685	738
	+69	10	24	34	62	244	303	732	766
13 30 to 14 15	+65	20	42	36	52	154	173	640	837
	66	15	32	30	41	172	198	695	834
	67	20	46	33	42	164	210	775	888
	68	17	44	34	53	195	242	723	775
	+69	16	32	39	57	202	279	646	698
14 15 to 15 0	+65	25	52	38	59	141	208	831	902
	66	15	40	36	66	204	254	808	897
	67	14	34	31	49	189	236	768	874
	68	8	37	20	50	141	245	641	688
	+69	18	41	28	75	134	283	672	758
15 0 to 15 45	+65	12	42	31	87	166	255	972	1039
	66	25	39	39	86	212	330	1024	1133
	67	19	42	46	54	238	286	949	1142
	68	13	37	29	59	210	251	685	796
	+69	16	33	38	63	215	273	742	842
15 45 to 16 30	+65	26	49	49	76	175	245	1201	1322
	66	14	38	43	75	233	290	1107	1205
	67	13	29	44	67	259	333	1112	1216
	68	16	29	37	64	268	308	999	1035
	+69	16	36	42	66	252	295	906	943

Limits of R.A.	Zone.	No. in B.D.		Number shown on Photographs. Exposure 20s.		Exposures 3m and 6m.		Exposure 40m.	
		9m·0 and brighter.	Total No.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.	Shown in duplicate.	Total No. of Stars.
h m	°								
16 30 to 17 15	+65	14	48	34	69	202	279	1190	1284
	66	19	52	53	81	251	317	1177	1287
	67	24	52	57	76	240	273	1368	1454
	68	18	43	37	65	259	301	1296	1338
	+69	23	51	62	90	247	316	1111	1209
17 15 to 18 0	+65	16	59	78	107	291	398	1408	1527
	66	18	62	54	82	301	367	1537	1935
	67	23	44	59	72	341	382	1839	1977
	68	24	48	64	93	372	397	1487	1548
	+69	23	61	68	111	338	392	1517	1546
18 0 to 18 45	+65	30	61	58	83	281	315	1697	2010
	66	18	58	72	94	316	403	1665	1744
	67	18	49	60	85	340	411	1359	1438
	68	18	51	46	87	310	446	1557	1753
	+69	15	37	50	89	346	469	1832	2051
18 45 to 19 30	+65	28	68	71	101	370	471	2170	2389
	66	24	84	85	121	442	545	2307	2441
	67	17	67	40	63	343	435	2249	2420
	68	17	51	57	102	406	479	2439	2534
	+69	17	51	97	131	538	608	1756	1868
19 30 to 20 15	+65	24	70	45	68	256	352	1979	2146
	66	27	72	37	69	256	390	1941	2022
	67	26	74	53	74	215	320	1300	1431
	68	15	49	41	58	290	326	1206	1296
	+69	16	42	43	56	237	253	1063	1162
20 15 to 21 0	+65	18	82	69	90	325	408	1792	2107
	66	23	78	37	79	209	307	1720	1816
	67	17	48	35	50	183	245	1003	1044
	68	17	57	47	78	322	378	1351	1499
	+69	15	44	55	88	307	358	1384	1616
21 0 to 21 45	+65	59	122	103	185	483	634	1910	1993
	66	34	96	46	78	256	352	1592	1749
	67	25	70	34	67	295	431	1535	1601
	68	21	74	57	97	391	441	1570	1773
	+69	21	62	52	82	314	374	1607	1686

Limits of R.A. h m	Zone. °	No. in B.D. 9m·0 and brighter.	No. in B.D. Total No.	Number shown on Photographs. Exposure 20s. Shown in duplicate.	Exposure 20s. Total No. of Stars.	Exposures 3m and 6m. Shown in duplicate.	Exposures 3m and 6m. Total No. of Stars.	Exposure 40m. Shown in duplicate.	Exposure 40m. Total No. of Stars.
21 45 to 22 30	+65	40	126	68	146	388	673	1761	2495
	66	34	93	94	177	563	888	2249	2424
	67	35	102	89	143	614	914	3015	3158
	68	22	62	78	123	553	654	3504	3754
	+69	35	70	96	134	480	569	1763	1908
22 30 to 23 15	+65	37	101	68	98	354	389	1521	1861
	66	30	84	50	72	305	348	1605	1681
	67	27	62	36	64	283	349	1284	1353
	68	22	70	63	95	362	442	1312	1477
	+69	15	47	43	98	345	443	1560	1870
23 15 to 0 0	+65	42	119	46	70	228	274	1107	1255
	66	32	84	47	84	268	420	947	1162
	67	32	86	48	79	286	461	1146	1396
	68	21	66	49	71	221	275	1168	1408
	+69	19	54	48	79	348	462	2029	2591
Totals ...		3,094	8,152	6,663	11,018	38,262	49,014	199,776	229,426
Number per square degree.		4·5	11·8	9·7	16·0	55·6	71·2	290·1	333·2
Ratio of Totals.		1	2·6	2·1	3·6	12·4	15·8	64·6	74·2

By taking the sums for the five zones the number of stars is obtained for each area of 5° in Decl., and 45m in R.A. between Decl. +65° and +70°. Each trapezium into which this belt of the sky is thus divided contains 21·52 square degrees.

Table II.

Number of Stars shown with various Exposures for Areas of 5° *in Declination and* 45m *in Right Ascension* (21·52 *square degrees*) *between* 65° *and* 70° *N. Decl.*

Limits of R.A. h m	h m	Number in B.D. 9m·0 and brighter.	Number in B.D. Total No.	Number shown on Photographs. Exposure 20s. Shown in Duplicate.	Exposure 20s. Total No. of Stars.	Exposures 3m and 6m. Shown in Duplicate.	Exposures 3m and 6m. Total No. of Stars.	Exposure 40m. Shown in Duplicate.	Exposure 40m. Total No. of Stars.
0 0–	0 45	133	364	241	404	1267	1493	6056	7098
0 45–	1 30	130	363	232	453	1168	1492	8080	9155
1 30–	2 15	100	286	179	315	1078	1309	5970	6618
2 15–	3 0	101	254	184	339	869	1144	4613	5531

Limits of R.A.				Number in B.D.		Number shown on Photographs.					
						Exposure 20s.		Exposures 3m and 6m.		Exposure 40m.	
h	m	h	m	9m·0 and brighter.	Total No.	Shown in Duplicate.	Total No. of Stars.	Shown in Duplicate.	Total No. of Stars.	Shown in Duplicate.	Total No.of Stars.
3	0–	3	45	94	258	170	306	929	1258	5047	6558
3	45–	4	30	84	245	140	244	759	1023	5799	6669
4	30–	5	15	109	239	184	305	1260	1606	10,028	12,209
5	15–	6	0	72	199	184	326	1265	1761	8581	11,181
6	0–	6	45	69	184	197	295	1336	1618	7570	9906
6	45–	7	30	82	193	143	246	1027	1403	6565	7773
7	30–	8	15	77	187	148	265	1031	1281	5510	6910
8	15–	9	0	75	210	173	300	974	1222	4382	4864
9	0–	9	45	76	200	137	210	713	842	3683	4391
9	45–	10	30	73	189	137	221	758	917	3902	4564
10	30–	11	15	65	161	171	250	858	1039	4044	4355
11	15–	12	0	80	172	167	282	957	1245	3836	4231
12	0–	12	45	66	157	129	220	746	985	3605	4006
12	45–	13	30	65	154	145	242	872	1048	3724	4096
13	30–	14	15	88	196	172	245	887	1102	3479	4032
14	15–	15	0	80	204	153	299	809	1226	3720	4119
15	0–	15	45	85	193	183	349	1041	1395	4372	4952
15	45–	16	30	85	181	215	348	1187	1471	5325	5721
16	30–	17	15	98	246	243	381	1199	1486	6142	6572
17	15–	18	0	104	274	323	465	1643	1936	7788	8533
18	0–	18	45	99	256	286	438	1593	2044	8110	8996
18	45–	19	30	103	321	350	518	2099	2538	10,921	11,652
19	30–	20	15	108	307	219	325	1254	1641	7489	8057
20	15–	21	0	90	309	243	385	1346	1696	7250	8082
21	0–	21	45	160	424	292	509	1739	2232	8214	8802
21	45–	22	30	166	453	425	723	2598	3698	12,292	13,739
22	30–	23	15	131	364	260	427	1649	1971	7282	8242
23	15–	0	0	146	409	238	383	1351	1892	6397	7812
Total ...				3094	8152	6663	11,018	38,262	49,014	199,776	229,426

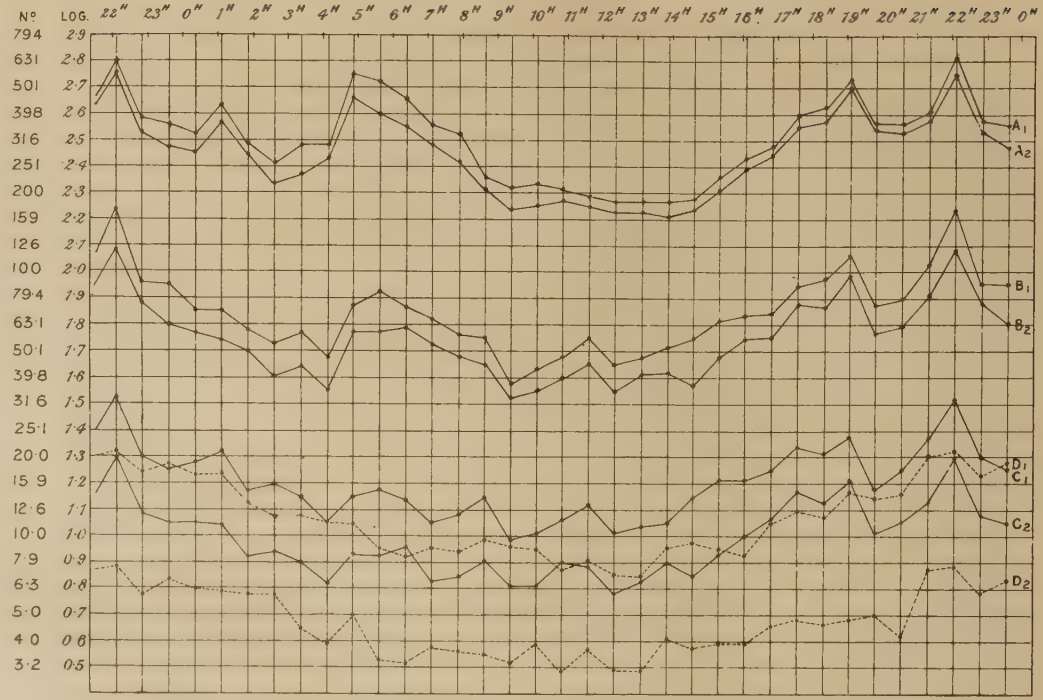
In Table III. the logarithms of these numbers are given, and in the diagram the logarithm of the number of stars per square degree is given, the numbers of the diagram being obtained from Table III. by subtracting logarithm 21·5 or 1·332.

TABLE III.

Logarithm of Number of Stars shown with Various Exposures for Areas of 5° in Decl. and 45^m in R.A. between the Limits +65° and +70° of Decl.

Limits of R.A.				Number in B,D.		Number shown on Photographs.					
						Exposure 20^s.		Exposures 3^m and 6^m.		Exposure 40^m.	
h	m	h	m	9^m·0 and brighter.	Total No.	Shown in Duplicate.	Total No.	Shown in Duplicate.	Total No.	Shown in Duplicate.	Total No.
0	0–	0	45	2·124	2·561	2·382	2·606	3·103	3·174	3·782	3·851
0	45–	1	30	2·114	2·560	2·366	2·656	3·067	3·174	3·907	3 962
1	30–	2	15	2·000	2·456	2·253	2·498	3·033	3·117	3·776	3·821
2	15–	3	0	2·004	2·405	2·265	2·530	2·939	3·058	3·664	3·743
3	0–	3	45	1·973	2·412	2·230	2·486	2·968	3·100	3·703	3·817
3	45–	4	30	1·924	2·389	2·146	2·387	2·880	3·010	3·763	3·824
4	30–	5	15	2·037	2·378	2·265	2·484	3·100	3·206	4·001	4·087
5	15–	6	0	1·857	2·299	2·265	2·513	3·102	3·246	3·934	4·048
6	0–	6	45	1·839	2·265	2·294	2·470	3·126	3·209	3·879	3·996
6	45–	7	30	1·914	2·286	2·155	2·391	3·012	3·147	3·817	3·891
7	30–	8	15	1·887	2·272	2·170	2·423	3·013	3·107	3·741	3·839
8	15–	9	0	1·875	2·322	2·238	2·477	2·989	3·087	3·642	3·687
9	0–	9	45	1·881	2·301	2·137	2·322	2·853	2·925	3·566	3·643
9	45–	10	30	1·863	2·276	2·137	2·344	2·880	2·962	3·591	3·659
10	30–	11	15	1·813	2·207	2·233	2·398	2·933	3·017	3·607	3·639
11	15–	12	0	1·903	2·236	2·223	2·450	2·981	3·095	3·584	3·626
12	0–	12	45	1·819	2·196	2·111	2·342	2·873	2·993	3·557	3·603
12	45–	13	30	1·813	2·188	2·161	2·384	2·940	3·020	3·571	3·612
13	30–	14	15	1·945	2·292	2·235	2·389	2·948	3·042	3·541	3·605
14	15–	15	0	1·903	2·310	2·185	2·476	2·908	3·088	3·570	3·615
15	0–	15	45	1·929	2·286	2·262	2·543	3·017	3·145	3·641	3·695
15	45–	16	30	1·929	2·258	2·332	2·542	3·074	3·168	3·726	3·757
16	30–	17	15	1·991	2·391	2·386	2·581	3·079	3·172	3·788	3·818
17	15–	18	0	2·017	2·438	2·509	2·667	3·216	3·287	3·891	3·931
18	0–	18	45	1·996	2·408	2·456	2·641	3·202	3·310	3·909	3·954
18	45–	19	30	2·013	2·506	2·544	2·714	3·322	3·404	4·038	4·066
19	30–	20	15	2·033	2·487	2·340	2·512	3·098	3·215	3·874	3·906
20	15–	21	0	1·954	2·490	2·386	2·585	3·129	3·229	3·860	3·907
21	0–	21	45	2·204	2·627	2·465	2·707	3·240	3·349	3·914	3·945
21	45–	22	30	2·220	2·656	2·628	2·859	3·415	3·568	4·090	4·138
22	30–	23	15	2·117	2·561	2·415	2·630	3·217	3·295	3·862	3·916
23	15–	0	0	2·164	2·612	2·377	2·583	3·131	3·277	3·806	3·893
Mean ...				1·970	2·385	2·298	2·518	3·056	3·162	3·769	3·828

Diagram giving number of Stars per square degree shown on Plates with various exposures, between 65° and 70° North Declination.



A_1 Total number of Stars shown with an exposure of 40^m.
B_1 Total number of Stars shown with exposures of 3^m & 6^m.
C_1 Total number of Stars shown with an exposure of 20^s.
D_1 Total number shown in Bonn Durchmusterung.

A_2 Number shown in duplicate, *i.e.*, on both of the overlapping places.
B_2 Number shown in duplicate, *i.e.*, on both of the overlapping places.
C_2 Number shown in duplicate, *i.e.*, on both of the overlapping places.
D_2 Number in B.D. of magnitude 9·0 and brighter.

Examination of the tables and diagram (Plate 7) shows the following results:—

(i.) The logarithm of the ratio of the greatest to the least number of stars per square degree is

0·54 or 0·55 for the 40^m exposure
0·56 or 0·64 for the 3^m and 6^m exposures
0·51 or 0·52 for the 20^s exposures
and 0·47 for the Bonn *Durchmusterung*.

where the first figure in each case is derived from the number of stars counted in duplicate and the second from the total number of stars. These figures show that the maximum number of stars per square degree rises to 3·5 times the minimum number for the 40^m exposure, about 4·0 times for the 3^m and 6^m exposures, and to about 3·2 times for the 20^s exposures.

(ii.) The numbers of the stars for the exposures 40^m, 6^m and 3^m, and 20^s maintain the same ratio for different parts of the sky. The only notable exception is the increase in the numbers of the 6^m and 3^m exposures, and more still of the 40^m exposures from 5^h to 6^h. This is not shown by the 20^s exposures.

(iii.) The variation in the number of stars per square degree given in the Bonn *Durchmusterung* is on the whole well supported by the short-exposure photographs. There is not in this region any marked falling off in the B.D. where the stars are rich, though there appears to be a slight deficiency at the maximum near 22^h. Relatively to the photographs the B.D. is rich in stars from 23^h to 5^h, and poor from 12^h to 19^h.

(iv.) Generally the number of stars shown with 20^s exposure is considerably in excess of the total number given in the Bonn *Durchmusterung*, while the number shown in duplicate (on both of the two overlapping plates) is generally less.

(v.) The ratio of the total number of stars shown to the number which are shown in duplicate on both overlapping plates is very constant for each of the three different series of exposures, but apparently differs from one to another. These ratios are :

Exposure.	Log. Ratio.	Ratio.
20^s	·220	1·66
3^m and 6^m	·106	1·28
40^m	·059	1·15

The cause of the smaller ratio for the 3^m and 6^m exposures is, as will be shown in the next paragraph, that the figures and diagrams given for the 6^m and 3^m exposures in the case of the "total number" of stars nearly correspond to the 3^m images, but do not for the number "shown in duplicate."

For the 40^m exposure the smallness of the ratio is probably due to the caution of the observers, who were instructed not to include any doubtful images seen only on one plate unless they were absolutely convinced that these were not photographic defects.

As it seemed of interest to obtain a formula giving the number of stars per square degree in terms of the duration of the exposure, a further analysis has been made of the number of stars shown with 3^m exposure for Zone 69°. These numbers are given in the following table compared with the corresponding numbers of Table I. The logarithms of their ratios are also given.

Number of Stars in Zone 69° *shown with an Exposure of* 3^m *compared with the Number previously tabulated of those shown with* 3^m *and* 6^m *Exposures.*

Limits of R.A.	Total Number. Exposure		Log. Ratio.	Number. Exposure		Log. Ratio.
h m h m	3^m or 6^m.	3^m.		3^m or 6^m.	3^m.	
0 0– 0 45	373	347	·031	302	199	·181
0 45– 1 30	413	399	·015	332	226	·167
1 30– 2 15	314	304	·014	255	186	·137
2 15– 3 0	420	408	·012	361	277	·116
3 0– 3 45	215	208	·014	187	152	·090
3 45– 4 30	228	217	·022	160	96	·222
4 30– 5 15	425	391	·036	361	250	·160
5 15– 6 0	375	360	·018	292	220	·123
6 0– 6 45	332	323	·012	275	199	·140
6 45– 7 30	232	227	·009	198	148	·127
7 30– 8 15	206	204	·004	169	137	·091
8 15– 9 0	271	268	·005	206	163	·102
9 0– 9 45	207	200	·015	170	118	·158
9 45–10 30	205	201	·009	144	96	·176
10 30–11 15	215	212	·006	173	122	·152
11 15–12 0	346	341	·006	249	207	·080
12 0–12 45	291	283	·012	217	174	·095
12 45–13 30	303	297	·009	244	190	·108
13 30–14 15	279	275	·006	202	138	·165
14 15–15 0	283	282	·002	134	105	·106
15 0–15 45	273	268	·008	215	158	·133
15 45–16 30	295	290	·008	252	209	·081
16 30–17 15	316	312	·006	247	209	·073
17 15–18 0	392	379	·014	338	266	·104
18 0–18 45	469	463	·005	346	281	·090
18 45–19 30	608	592	·012	538	419	·109
19 30–20 15	253	248	·009	237	172	·139
20 15–21 0	358	346	·015	307	231	·123
21 0–21 45	374	356	·022	314	221	·153
21 45–22 30	569	551	·014	480	392	·088
22 30–23 15	443	438	·005	345	261	·121
23 15– 0 0	462	456	·006	348	264	·120
Mean			·012			·126

The above table shows that the total number of stars which give images with 3^m exposure on one of the two plates for each area is only very slightly less than the total number given in Tables I., II., III., and in the diagram as corresponding to an exposure of 3^m or 6^m, the difference ·012 of the logs. corresponding to a ratio of 0·97 to 1. This is due to the method adopted in the measurement of rejecting single images seen only on one plate (see p. 121). There is, however, a very considerable difference in the number of stars shown on both plates with an exposure of 3^m and the corresponding numbers tabulated above for 3^m and 6^m; the difference of ·126 in the logarithms corresponding to a ratio of ·75 to 1·00.

Assuming that these figures, which are actually found only for Zone 69°, apply to the five zones, we have for the logarithms of the number of stars per square degree with the different exposures—

I. *Shown on both Plates.*

20^s.	3^m.	6^m.	40^m.
0·966	1·598	1·724	2·437

II. *Total Number shown.*

20^s.	3^m.	6^m.	40^m.
1·186	1·818	1·830	2·496

It would seem that these numbers are accurate in the first case, but that in the second case not all the stars shown with 6^m exposure are counted, and probably not all of those shown with an exposure of 40^m. The difference of the logarithms of the number of stars shown with an exposure of 3^m and with an exposure of 20^s is ·632 both for the number of stars shown in duplicate and for the total number, while the difference for the exposures 40^m and 20^s is 1·471 for the stars shown in duplicate and 1·310 for the total number. Assuming the formula $\log \frac{N}{N_o} = k \log \frac{T}{T_o}$, where N is the number of stars per square degree shown with duration of exposure T, the interval 20^s to 3^m gives $k = ·66$, and the interval 20^s to 40^m gives $k = ·70$ for stars shown in duplicate and ·63 for the total number. Thus we may take $\log \frac{N}{N_o} = ·67 \log \frac{T}{T_o}$ or $\left(\frac{N}{N_o}\right)^2 = \left(\frac{T}{T_o}\right)^3$ as a good approximate formula for the increase of numbers with exposure between the limits of 20^s and 40^m.

On the assumption that an equal total amount of light produces an equal photographic effect, an additional magnitude is reached by increasing the exposure 2·5 times. Between the 3^m and 20^s (ratio of 9 to 1) there corresponds a difference of magnitude $2^m·36$. Between the 40^m and 20^s exposure (ratio 120 to 1) there is a difference of $5^m·20$. If r be the ratio of the

number of stars down to magnitude $m + 1$ to the number down to magnitude m, we obtain

$r^{2\cdot38} = 4\cdot29$ [log = ·632] from the 20^s and 3^m images
$r^{5\cdot20} = 29\cdot58$ [log = 1·471] from the 20^s and 40^m images

Thus from the 20^s and 3^m images we get $r = 1\cdot84$
and from the 20^s and 40^m images we get $r = 1\cdot92$

These ratios are deduced from the number of stars shown in duplicate.

The variation of the star density with the distance from the Milky Way is distinctly shown, though not in very striking manner by these counts, as the area considered only just reaches the Milky Way at about 0^h, and is 50° from it at 12^h. The minimum number of stars per square degree, especially for the 40^m exposure, is very noticeable from 9^h to 15^h. Here the distance from the Milky Way lies between 50° to 43°. There is a gradual rise to about three times this number of stars at 5^h and 19^h, when the centre of the zone considered approaches closely to the Milky Way, and there is a still more pronounced maximum at 22^h, which is strikingly shown with all three exposures. Between 19^h and 5^h, where the zone approaches and just enters the galaxy, there are, as the diagram shows, some irregularities with two maxima and three minima.

Note on the Reproduction and Publication of the Photographs for the Astrographic Chart taken at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

In accordance with the recommendation of the International Committee, it is proposed to reproduce and distribute to a limited number of Observatories and Institutions the Greenwich portion of the Astrographic Chart, the original negatives being enlarged to twice the scale.

After careful trial a method of direct photographic reproduction has been adopted, prints being taken by contact on bromide card (15 in. × 12 in.) from an enlarged positive on glass. By this method it is found that practically no stars are lost in reproduction, and that there is little liability to the introduction of false stars in the process beyond those which may exist in the original negative, and these can readily be detected by reference to the overlapping plate. They are for the

most part large and round, and can be as readily distinguished with a single image as with a triple image, which for such large stars is practically blended into one.

On full consideration it is believed that sufficient provision will be made for the utilisation of the Astrographic Chart if the reproduction is limited to fifty sets, to be distributed to a selected list of Observatories and Institutions (including the eighteen participating Observatories) where proper provision is likely to be made for the satisfactory storage of the 22,000 sheets of which the chart will consist ; an important consideration in view of the bulky nature of the work. It is to be noted that additional sets can be taken subsequently to meet any further demand which may arise.

A careful estimate gives for the cost of reproduction of fifty sets of the 1,149 plates of the Greenwich Zone, including the preparation of two sets of positive enlargements on glass, 3,000*l.*, or at the rate of 60*l.* a set—equivalent to one shilling per print on bromide card 15 in. × 12 in. This is the total cost of labour, plates, paper, and chemicals. It is estimated that two persons working together can complete the fifty sets for the Greenwich Zone (about 2,400 positive enlargements and 60,000 bromide prints, 15 in. × 12 in.) in from five to six years.

The following method has been adopted as the result of experiments on purely photographic lines. From the original chart plate a positive on glass of twice the scale is made in a camera using a lens working at *f*/8 and with a focal length of 11·2 inches. A uniform illumination is obtained by the irregularly reflected light of a white screen, illuminated by electric lamps arranged symmetrically round the plate.

From this positive, prints on glazed bromide paper are made by contact. The paper employed is a thin card, the sheets being 15 inches by 12. This gives a half-inch margin at the sides and about 2 inches at the top and bottom, thus leaving room for the insertion of title and data concerning the plate.

The prints obtained by this process are exact copies of the original, and no retouching is effected to eliminate any defects which might be mistaken for stars. Generally there is no difficulty, but in all cases of doubt reference to the same field on the overlapping plate can be made. Counts were made of the number of stars shown on the original negatives and on the prints for three pairs of overlapping plates, and 1,046 images were found on the prints and 1,060 on the original plates. It thus appears that practically all stars shown on the plates are reproduced on the prints.

Note on Photographs of Comet d 1902 *Giacobini obtained with the 30-inch Reflector at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

Seven photographs of this comet have been obtained with the 30-inch reflector on six nights, with exposures varying from thirty minutes to five minutes. In the case of three photographs the driving of the telescope was corrected to allow for the comet's motion, but the remaining photos were taken with the telescope guided on the stars, the comet's motion in ten minutes being very small. The comet is well shown with an exposure of ten minutes, the photographic image being of stellar character, and capable of very exact measurement.

The following is a list of the photographs obtained up to the present time :—

Date.	Exposure.	Remarks.
1902.		
Dec. 29	30^m	Comet's motion in R.A. and Decl. corrected.
,, 30	10^m, 15^m	Comet's motion in Decl. corrected.
,, 31	10^m, $7\frac{1}{2}^m$, 10^m, 5^m	,, ,,
1903.		
Jan. 2	10^m, 10^m, 8^m	Telescope guided on the stars.
,, 3	10^m, 10^m, 10^m	,, ,,
,, 7	15^m	,, ,,
,, 7	15^m	,, ,,
,, 8	10^m, 10^m	,, ,,

Royal Observatory, Greenwich :
1903 *January* 9.

Note on Plate Constants. By F. W. Dyson, M.A., F.R.S.

In a paper by the Astronomer Royal and myself (*Monthly Notices*, vol. lvi. pp. 114–134) the reduction of a number of astrographic plates according to the formulæ and methods given by Professor Turner (*Monthly Notices*, vol. liv.) was considered. It was shown that the constants a and e computed in this way gave discordances with the values of these constants calculated so as to correct for the scale of the telescope, and for the differential effects of aberration and refraction of a magnitude which might be expected from the errors of the right ascensions and declinations of the reference stars. Similarly the constants b and d, which are mainly caused by an orientation error of the plate, showed differences of the magnitude which would arise from the errors of the reference stars. It was considered, therefore, that it would be better to use calculated values of a and e,

and a value of the orientation error from the mean of the b and d results, and thus use the data of the plate to furnish only three constants—namely, the orientation and the errors of the centre of the plate in the two directions. This was a modification of Professor Turner's method, in which six independent constants were derived from the data of the plate itself. The grounds for the use of six constants are that these will tend to correct for possible errors of distortion, whether optical or arising during the drying of the gelatine, tilt of the plate, &c. The conclusion which was arrived at by the Astronomer Royal and myself was that the uncertainties of the position of the reference stars were sufficient to account for the differences of the plate constants from their theoretical values.

The six plate constants have been computed by Professor Turner's method for 508 of the Greenwich astrographic plates. For 376 of these the right ascensions and declinations of the Helsingfors and Christiania Catalogues have been used. These stars are being reobserved at Greenwich; and, although not more than two observations have been generally obtained, it seemed of interest to recompute with the positions of the reference stars the plate constants for those plates which showed the greatest discordances.

The following table gives the results of the two determinations (the unit being in the fifth decimal place, *i.e.* −99 is −·00099, &c.):—

TABLE I.

Plate Constants.

No. of Plate.	No. of Stars.	Old Determination.				New Determination.			
		a.	*c.*	*b.*	*d.*	*a.*	*c.*	*b.*	*d.*
336	21	− 57	− 41	+133	−180	− 85	− 65	+166	−159
807	16	− 48	−110	+296	−306	− 61	− 66	+279	−314
862	11	− 6	− 57	+281	−320	− 56	− 79	+305	−282
938	14	− 53	−126	+105	−108	− 68	− 80	+101	−124
941	12	− 27	− 90	+ 80	− 60	− 60	− 71	+ 75	− 69
1610	18	− 99	− 68	+120	−190	− 57	− 76	+137	−158
1762	17	− 96	− 40	+407	−428	− 87	− 74	+373	−414
2042	22	− 77	−137	−107	+129	− 92	− 81	−109	+138
2044	21	− 95	− 77	+ 36	− 98	− 99	− 77	+ 40	− 65
2048	28	− 54	− 32	+ 45	− 10	− 59	− 46	+ 20	− 4
2328	17	−103	− 37	+122	−130	−105	− 95	+138	−159
2568	25	− 30	− 85	+102	−105	− 55	− 85	+102	−106
2685	20	− 66	− 82	+220	−282	− 78	− 81	+235	−244
2859	41	− 74	− 77	+242	− 308	− 70	− 76	+293	−300
2981	15	− 35	−113	+370	−338	− 76	− 66	+373	−371
2995	9	− 57	− 33	+294	−303	− 67	− 63	+309	−332
4807	18	− 67	− 28	+ 77	− 71	− 78	− 76	+ 19	− 31

It will be seen that all the large discordances in the values of a and e, whose theoretical value is about -00070 (varying principally owing to aberration from -00064 to -00076) are removed.

The discordances between the plate constants are so much diminished by improved right ascensions and declinations of the reference stars that it seems reasonable to look upon the uncertainties in the positions of these stars as the main source of error, and to conclude that any real variations of the constants due to distortion, &c., are less than the accidental errors of their determination. This is at any rate the case unless the reference stars are extremely well observed ; and less error will be introduced by neglecting the small possible effects of distortion, tilt, &c., than by introducing the accidental errors which are involved in the determination of three additional constants.

The large difference in the values of b and d in the last plate, in addition to the change in the value of e, made it of interest to compute the final residuals with the adopted constants $a = e = -\cdot00075$ $b = -d = +25$. The following table gives the right ascensions originally used, the right ascensions and declinations from recent Greenwich observations, and the differences between Greenwich and Christiania, and between the Greenwich meridian observations and the photographs. The numbers in the third column give the number of observations at Greenwich.

TABLE II.

(*Assumed right ascensions and declinations of reference stars on Plate* 4807 *in the first and second determinations of plate constants, and the final residuals with the adopted constants* $a = e = -\cdot00075$ $b = -d = +\cdot00025$.)

No. in Chr. Cat.	Position from Greenwich Observations. R.A.		Decl.	No. of Obs.	Corrections to Christiania. R.A.	Corrections to Christiania. Decl.	Final Residuals on Photo. R.A.	Final Residuals on Photo. Decl.
	h m s		° ′ ″		s	″	″	″
1567	9 57 51·93		68 4 37·4	3	−·01	−0·0	+0·5	−0·4
51	49 12·97		14 12·5	2	+·13	+0·7	+0·5	+0·2
66	57 34·62		23 5·8	2	−·07	−0·6	−0·5	−1·1
41	45 30·49		38 14·5	2	+·38	+2·1	−0·7	+0·4
32	39 11·82		56 50·1	2	−·14	−0·7	−0·5	+0·7
52	49 18·74		57 19·3	2	−·08	−0·4	+0·3	+0·4
79	10 1 33·70		59 9·8	1	−·11	−0·6	+1·2	−0·3
68	9 58 23·15		69 8 57·9	2	−·07	−0·4	+0·3	0·0
56	53 28·74		11 52·4	3	−·08	−0·5	+0·3	−0·1
77	10 0 56·22		11 29·9	2	−·21	−1·1	−0·7	−0·2
80	1 37·80		9 13·9	1	−·20	−1·1	−1·4	−0·2
59	9 54 53·81		16 3·3	3	−·34	−1·7	−0·6	+1·2
44	46 47·25		24 42·8	1	−·45	−2·4	0·0	−0·4
45	46 49·08		24 42·6	1	−·58	−3·0	+0·5	−0·2

No. in Chr. Cat.	Position from Greenwich Observations. R.A.	Decl.	No. of Obs.	Corrections to Christiania. R.A.	Decl.	Final Residuals on Photo. R.A.	Decl.
	h m s	° ′ ″		s	″ ″	″	″
46	46 49·97	22 21·1	5	+·02	+0·1 −1·5	+0·8	+0·1
50	48 47·58	22 28·0	2	+·20	+1·1 −0·2	−0·4	−0·6
58	54 20·45	31 16·3	4	−·27	−1·4 +0·6	−0·1	+0·1
42	46 8·34	55 21·7	1	−·61	−4·2 −4·0	+0·5	−0·8

Taking this plate, which was chosen quite at random, as a fair sample of the results which will be given by the Greenwich astrographic work, the smallness of the residuals is extremely satisfactory.

In R.A. the mean discordance is $\pm$·0018 = $\pm$0″·54
In Decl. „ „ „ „ $\pm$·0014 = $\pm$0″·41

The corresponding probable errors are $\pm$0″·46 and $\pm$0″·35.

In the introduction to the Greenwich Catalogue for 1890 the probable accidental error of an observation of right ascension is given as $\pm$^{s}·033 (= $\pm$″·50), and between the limits of Decl. of the Greenwich Zone the probable accidental error of an observation of Decl. is $\pm$0″·46. Each of the reference stars on this plate has been observed about twice, so that the resulting R.A. and Decl. may be expected to have probable errors of about $\pm$0″·35 and $\pm$0″·32.

Comparing these with the figures $\pm$0″·46 and $\pm$0″·35, it is clear that the errors of the photographic results are extremely small.

The above table, as well as Table I., shows the advantage which has resulted from the re-observation of the reference stars with the transit circle. With five observations of each reference star it would seem the uncertainties of the plate constants, which are the principal source of the error in the deduction of R.A. and Decl. from the astrographic photographs, will be sufficiently reduced.

Returning now to the computation of the plate constants, there is a considerable simplification of the work when a and e are known beforehand. The quantities $x - ax$ are tabulated in order of y and equated to $by + c$. The solution of these equations, which should be done by least squares with approximate weights when the number of stars is small or when the stars are badly distributed, gives b and c. Similarly the quantities $y - ey$ are arranged in order of x and equated to $dx + f$, and the equations solved for d and f. The mean of b and $-d$, with a possible correction of ·00001 when the plate is taken one hour from the meridian, is adopted. The advantage of the arrangement in order of y and x is that any mistakes which may have been overlooked are likely to be detected before the equations are solved.

A Graphical Method of Applying to Photographic Measures the Terms of the Second Order in the Differential Refraction. Arthur R. Hinks, M.A.

1. The great and well-known advantage of Turner's methods of reducing in rectangular coordinates the measures made upon a photograph depends upon the fact that all the various small differences, due to refraction, aberration, orientation, centering, &c., by which the measures of stars on an actual plate differ from what they would be on an ideal "standard" plate are usually linear functions of the coordinates, and their value can be deduced *en bloc* by comparing the measured coordinates of known stars with their computed coordinates on the ideal plate. So soon as any of the corrections necessary begin to involve terms of the second order in x and y, the measured coordinates, the simplicity of the method is lost, but may be restored by finding separately and applying to the measures such parts of the whole reductions as are of the second order in x and y; when this is done the main part of the reduction proceeds as before.

2. The only parts of the reductions which are likely to involve second-order terms are the differential refractions. In the reduction of the photographs of *Eros*, many of which were necessarily taken at considerable zenith distances, these terms occasionally amount to one- or two-tenths of a second of arc; it is therefore necessary to take account of them before proceeding to form the equations and make the solution for the ordinary six constant reductions. But since nothing is more aggravating than having to compute for each star a number of small quantities which when added together amount generally to not more than one or two units in the last place of decimals employed, I tried various ways of computing these small terms graphically, and have succeeded in constructing three diagrams by means of which the second-order refraction terms can be found in a few minutes for any plate taken anywhere in the world. It seems possible, then, that these diagrams may be found of general use.

3. The expressions for the complete effect of refraction have been given by Professor Turner (*Monthly Notices*, lvii. 1897, p. 133). It is there shown that if (x, y) be the coordinates of an unrefracted star, and β the constant of refraction, the effect of refraction is to displace the star image on the plate to the point whose coordinates are

$$x+t(\mathrm{X}-x),\ y+t(\mathrm{Y}-y)$$

where

$$t=\beta\frac{1+x^2+y^2}{1+x\mathrm{X}+y\mathrm{Y}+\beta\{x(x-\mathrm{X})+y(y-\mathrm{Y})\}}$$

and X, Y are the coordinates of the projection on the plate of the zenith. It is further shown that the term in β in the denominator may be neglected.

If we expand the above expressions the whole refractions Δx, Δy are given by

$$\Delta x/\beta = X - x(1+X^2) - yXY + x^2X(2+X^2) + xyY(1+2X^2) + y^2X(1+Y^2)$$
$$\Delta y/\beta = Y - xXY - y(1+Y^2) + x^2Y(1+X^2) + xyX(1+2Y^2) + y^2Y(2+Y^2)$$

where the square of β and the cubes of x and y are neglected.

[It should be noted that Professor Turner has inadvertently omitted in his paper these general expressions. Those given on p. 136 are for the particular case Y = 0.]

The terms in the first line of each expression are the terms that are included in the ordinary reductions implicitly, together with the other linear expression for the corrections for aberration, &c. The second-order terms, in the second line, are those which I shall propose to find by a graphical method.

4. Suppose X = 0; that is, the plate is taken on the meridian.

Then $Y = \tan$ (zenith distance of centre)
$= Z$ suppose

and $\Delta x/\beta = Z \,.\, xy$
$\Delta y/\beta = Z \,.\, x^2 + Z(2+Z^2) \,.\, y^2$

In this particular case the axis of y, the north and south line on the plate, is also the projection of the vertical great circle through the centre of the plate; and it is clear that if we take a plate in any hour-angle, with centre at zenith distance $\tan^{-1}(Z)$, and refer our stars to rectangular axes of h, k, of which the axis of k is the projection of the vertical, then

$$\Delta h/\beta = Z \,.\, hk$$
$$\Delta k/\beta = Z \,.\, h^2 + Z(2+Z^2) \,.\, k^2$$

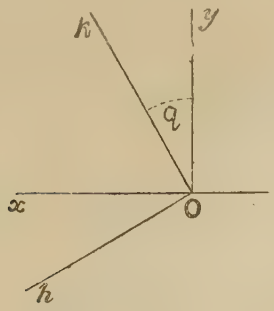


Fig 1.

We have now two sets of axes on the plate; Ox, Oy the axes of measurement of which the positive direction of Ox is towards increasing right ascension, the positive direction of Oy northwards; Oh, Ok, the new axes, of which Ok is the projection of the vertical, with positive direction towards the zenith, Oh at right angles to this, and positive towards the east.

The angle kOy is the parallactic angle at the point corresponding to the centre of the plate, and according to the usual convention increases positively as the hour-angle increases westward of the meridian—that is, as Ok swings round from positive Oy towards positive Ox.

Consequently the displacements in x and y, due to the second-order terms of the differential refraction, are

$$\Delta x/\beta = Zh^2 \sin q + Zhk \cos q + Z(2+Z^2)k^2 \sin q$$
$$\Delta y/\beta = Zh^2 \cos q - Zhk \sin q + Z(2+Z^2)k^2 \cos q$$

All the points on the plate for which the *displacement* in x is a quantity c (the *correction* $-c$) lie on the conic

$$c/\beta = Zh^2 \sin q + Zhk \cos q + Z(2+Z^2)k^2 \sin q,$$

which is at present referred to the axes of h, k. And a similar equation gives the locus of a given displacement in y.

5. To draw these curves we must refer them to their principal axes. Let Ou, Ov be the principal axes of the Δx conic; let Ov make an angle ϕ with the axis Ok, increasing in the same sense as the angle q.

Then, remembering that $Z \sin q = X$; $Z \cos q = Y$ we easily find that

$$\cot 2\phi = X(1 + X^2 + Y^2)/Y$$

and after considerable reduction the equation of the conic is found to be

$$(X - \tfrac{1}{2} Y \tan \phi)\, u^2 + (X + \tfrac{1}{2} Y \cot \phi)\, v^2 = c/\beta$$

If then a, b are the semi-axes of the conic

$$b = \left\{ \frac{c}{\beta (X + \tfrac{1}{2} Y \cot \phi)} \right\}^{\frac{1}{2}}; \quad \frac{a}{b} = \left\{ \frac{X + \tfrac{1}{2} Y \cot \phi}{X - \tfrac{1}{2} Y \tan \phi} \right\}^{\frac{1}{2}}$$

6. And similarly it may be shown that the locus of points where the *displacement* in y is c (and consequent *correction* $-c$) is a conic for which

$$b' = \left\{ \frac{c}{\beta (Y - \tfrac{1}{2} X \cot \phi')} \right\}^{\frac{1}{2}}; \quad \frac{a'}{b'} = \left\{ \frac{Y - \tfrac{1}{2} X \cot \phi'}{Y + \tfrac{1}{2} X \tan \phi'} \right\}^{\frac{1}{2}}$$

where $\cot 2\phi' = -Y(1 + X^2 + Y^2)/X$.

7. Now suppose that we had a diagram or a table of double entry, with arguments X, Y, from which to find ϕ; and that

we had found ϕ for certain values $X = m$, $Y = n$. It is clear that if we had taken $X = n$, $Y = m$, we should have found the value of $-\phi'$ instead of ϕ; and the same table or diagram will give us the values both of ϕ and of ϕ'.

Further, if we had a table or diagram to give us the value of b when X and Y are known, and had found b for $X = m$, $Y = n$; substituting instead $X = n$, $Y = m$, we should have found b' instead of b; and the same table or diagram will give us b and b' by an interchange of the values we assign to the arguments X and Y.

A similar interchange of arguments gives us either a/b or a'/b' from the same diagram.

Again, in a table which gives q with arguments X and Y, interchange of the values of the arguments gives us $\frac{\pi}{2} - q$ instead of q. And we have seen that in a table for ϕ interchange of the arguments gives us $-\phi'$. Hence in a table for $q + \phi$, interchange of the arguments gives us $\frac{\pi}{2} - (q + \phi')$, and a table of $\frac{\tan}{\cot}(q + \phi)$ will give us by interchange of the arguments $\frac{\cot}{\tan}(q + \phi')$. It follows that we need only three diagrams, for b, a/b, and $(q + \phi)$, all with double arguments X and Y, to enable us to draw both the Δx and the Δy conics.

8. Consider now the first of the Δx conics. It is to be the boundary line on the plate between the regions for which Δx is respectively 0 and 1 in the last place of decimals which we propose to use in our reductions, and the question arises, What shall we take as the unit of our measures in diagrams which may, it is hoped, be of general use? Were there no uniformity at all in the focal lengths of photographic telescopes, there could be, I imagine, no doubt that the most convenient unit in which to *publish* measures (not to make them) would be the focal length of the telescope, or some decimal part of it. The fact that there is actually considerable uniformity, due to the adoption of a standard pattern for the telescopes which are making the astrographic chart, and which give a scale $1' = 1$ mm., might suggest that it would be better to adopt as the unit either a minute of arc or five times that quantity, the astrographic réseau interval. Inasmuch, however, as there is no uniformity in the practice of the Observatories which are making the chart and catalogue, since some publish their measures in minutes of arc and some in réseau intervals; further, since it is necessary to go to four places of decimals in either case, though the ten-thousandth of $5'$ is rather too large, and the ten-thousandth of $1'$ decidedly too small to bear its proper proportion to the actual uncertainty of the last figure retained; and, finally, since in the *Eros* reductions which we have undertaken at Cambridge a considerable proportion of the measures, made in millimetres, on plates from Lick, Northfield, Minneapolis, and Cambridge,

have no simple relation to the minute of arc, I have decided to adopt as a unit the one-thousandth part of the focal length.

If this is our unit, one in the fourth place of decimals is 0″·02, which seems to me of just the right amount—about one-third of the P.E. of a complete measure on the very best plates. It is scarcely necessary to remark on the convenience of working in parts of the focal length, especially in work like photographic reductions of the present day, where the forms of the reductions are still subject to continual change, where one has to be always constructing new methods and formulæ, and where no single process is yet accepted as canonical.

9. Our first conic has to be the boundary of the zone on our plate where Δx begins to affect the fourth place of decimals—that is, where $\Delta x = 0{\cdot}00005$, the unit in which x is expressed being $10^{-3} \times$ (the focal length). But it will be found convenient in practice to leave X, Y in parts of the focal length, not of our new unit. We have then for the minor axis of the first conic in our new units

$$b = 10^3 \left\{ \frac{5 \times 10^{-8}}{\beta'' \sin 1'' \,(\mathrm{X} + \tfrac{1}{2}\,\mathrm{Y} \cot \phi)} \right\}^{\frac{1}{2}}$$
$$= 13{\cdot}3\,(\mathrm{X} + \tfrac{1}{2}\,\mathrm{Y} \cot \phi)^{-\frac{1}{2}}$$

whereas, if we are working in a unit whose ratio to the focal length is μ, we have in that unit

$$b = \frac{1}{\mu} \left\{ \frac{5\mu \times 10^{-5}}{\beta'' \sin 1'' \,(\mathrm{X} + \tfrac{1}{2}\,\mathrm{Y} \cot \phi)} \right\}^{\frac{1}{2}}$$
$$= 0{\cdot}420 \times \mu^{-\frac{1}{2}}\,(\mathrm{X} + \tfrac{1}{2}\,\mathrm{Y} \cot \phi)^{-\frac{1}{2}}$$

10. From the first of these expressions for b I have constructed diagram I. (Plate 8), from which the values of b can be taken, with an accuracy of one tenth of a unit, when X and Y have been found for the plate for which it is desired to construct the refraction diagram.

And, as has already been pointed out, the value of b', the minor axis of the first Δy conic, is found from the same diagram by interchanging the values of X and Y.

In a similar way diagram II. (Plate 9) gives the values of a/b and a'/b', the ratios of the axes of the two conics.

To draw the conics we must know their orientation with respect to the standard axes of measurement. The angles $q+\phi$, $q+\phi'$ are the angles between the minor axes of the two conics for Δx and Δy and the axis of y. If the diagram is drawn so that y increases positively upwards and x towards the left, then the direction of positive increase of $q+\phi$, $q+\phi'$ is counter-clockwise.

Diagram III. (Plate 10) gives the values of the tangent or cotangent of these angles, according as they are less than, or greater

DIAGRAM 1.

b and b'.

The sign of the correction is + or − according as $\begin{matrix}\text{X for } b\\ \text{Y for } b'\end{matrix}$ is − or +.

b and b' are given by this diagram in thousandths of the focal length. If it is required to express them in terms of another unit whose ratio to the focal length is μ, the values found here must be multiplied by $(1000\,\mu)^{-\frac{1}{2}}$.

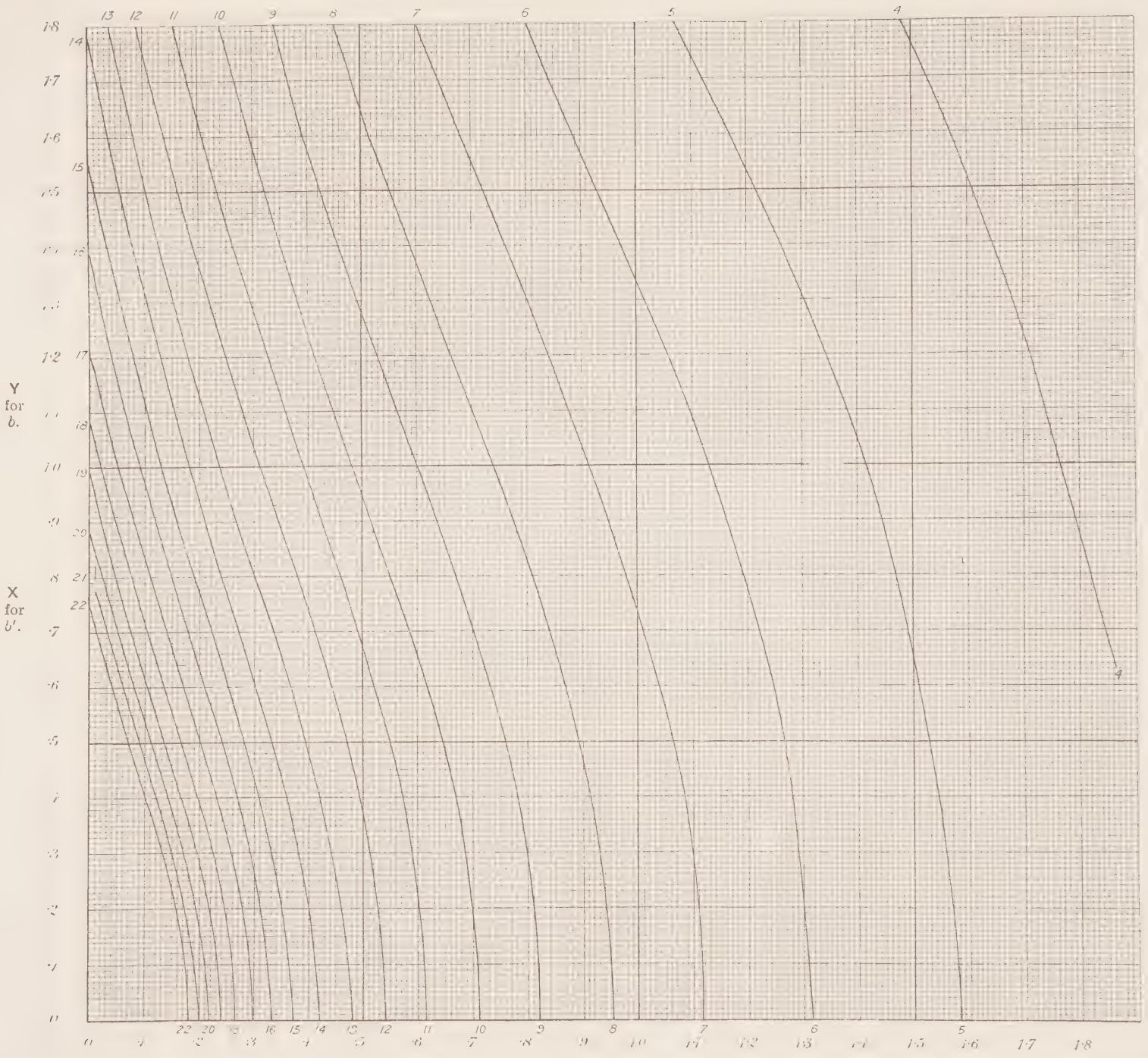


DIAGRAM II.

a/b and a'/b'.

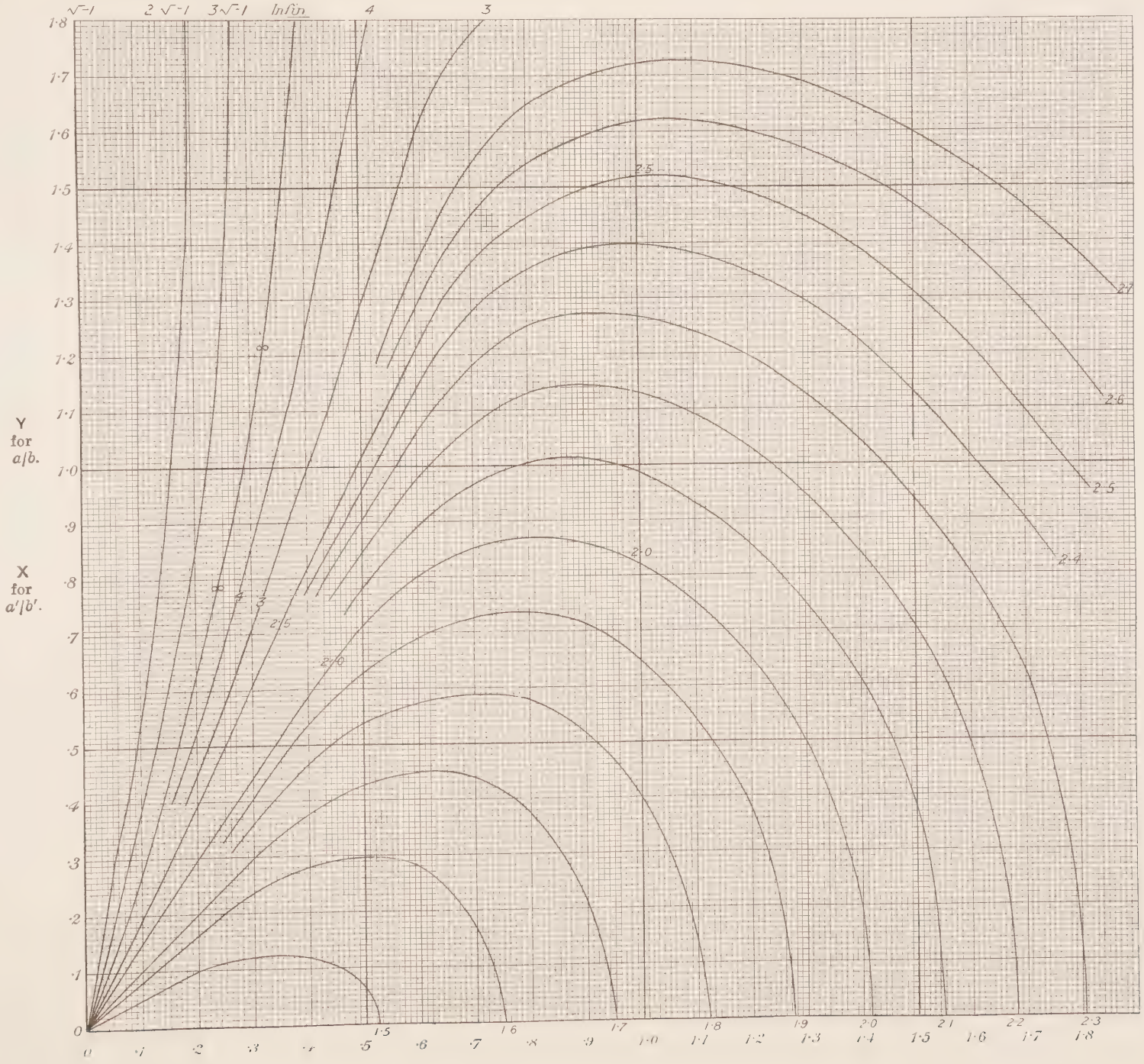
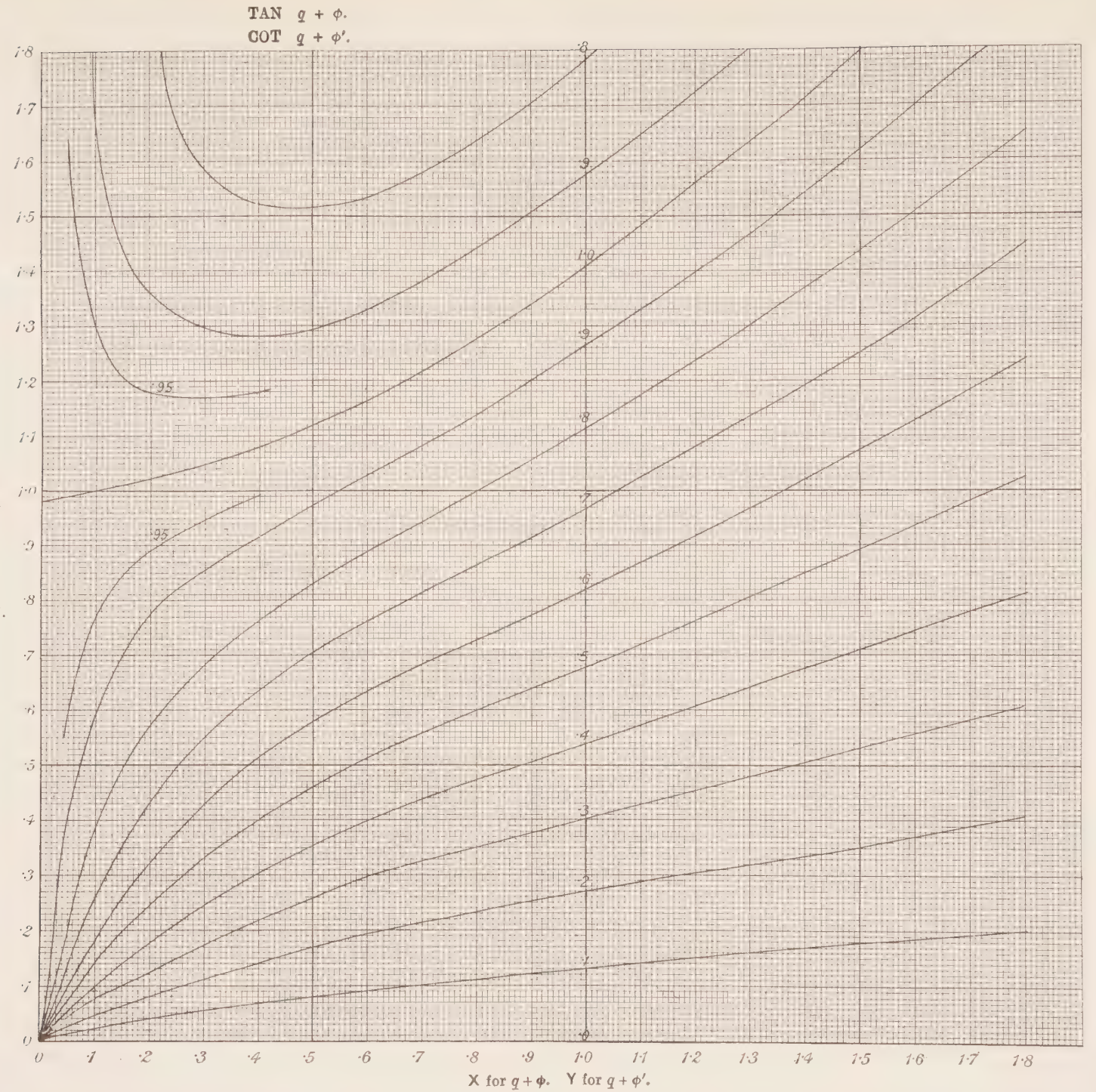


DIAGRAM III.

TAN
or COT $q+\phi, q+\phi'$.

If X and Y are of same sign, quantities from this diagram are + ve; if X and Y are of opposite sign, quantities are − ve.



than 45°, and with these quantities the axes of the conics may be laid down on squared paper without trouble.

11. In by far the greater proportion of cases the Δx and Δy conics are both ellipses. If either X or Y is small, one or the other of them will generally be an hyperbola ; but its semi-axes will usually be so large that it will lie altogether off the plate diagram, and we will postpone for a little the treatment of the few cases when the conic is an hyperbola and comes on to our plate diagram.

Consider a case where the first Δx conic is an ellipse, semi-minor axis b. This ellipse is the boundary between the regions on the plate where the second-order refraction correction Δx is respectively 0 and 1 in the fourth place of decimals ; or it is the locus of points where the correction is ·00005. The boundary between the zones of correction 1 and 2 will be a similar conic whose semi-axes are $\sqrt{3}$ times those of the first, and whose axes are in the same ratio. Similarly, between the zones of correction 2 and 3 the boundary is a conic with axes $\sqrt{5}$ times those of the first, and so on.

Now if we had to draw all these ellipses the method would be valueless in practice—at least, until someone shall invent an ellipsograph to draw ellipses of a required size and shape without troublesome adjustments. But, fortunately, all that is necessary is to draw the auxiliary circles upon the minor axes ; that is to say, having given b for the first ellipse, to draw a series of concentric circles with radii b, $\sqrt{3}b$, $\sqrt{5}b$, $\sqrt{7}b$, and so on, and to draw the lines representing their common axes from the data of diagram III. Now number the zones between successive pairs of circles 1, 2, 3, . . ., and mark on the diagram—which, of course, is drawn on squared paper—the position on the plate of a star for which Δx is required. Suppose the coordinates of the star, *referred to the axes of the ellipses*, are u, v. The point (u, v) lies inside or outside a certain ellipse, according as the point bu/a, v lies inside or outside the corresponding auxiliary circle. Hence, if we take a pair of proportional compasses set to the ratio a/b, it is a matter of a few seconds only to determine for each star between which pair of ellipses it lies, and consequently what the correction is for the second-order terms of the differential refraction.

Diagram I. gives b for any values of X and Y, without discriminating whether the conic is an ellipse or hyperbola. Diagram II. shows with a thick line the curve of $a/b=\infty$, which gives the boundary between the ellipses and hyperbolas. Occasionally it will be found that the first hyperbola of a set just comes on to the corners of a plate—that is to say, that the corresponding correction is just more than half one in the last place. If it is not considered legitimate to neglect so small a correction, it will at least be sufficient to sketch in the hyperbolas freehand ; and this I have done in the example which follows.

It will be noted that when we have an hyperbola bounding

a region of positive correction, the conjugate hyperbola bounds a region of equal negative correction, and must also be drawn in.

12. It does not seem to be necessary to go into details about the way the rules of signs have been found. The rules, as they are given in diagrams I.–III., follow almost immediately from the theory, and it is easier to deduce them afresh than to read an explanation of how they were deduced at first.

13. Let us take an example of the way in which, by the aid of diagrams I., II., and III., a refraction diagram can be drawn for any plate taken anywhere in the world.

Among the measures of photographs of *Eros* and comparison stars communicated to Cambridge by the kindness of the directors of several Observatories, is a plate taken at San Fernando, 1900 November 15, S.T. $22^h\ 15^m\ 27^s$. The approximate coordinates of its centre are

$$\text{R.A. } 1^h\ 47^m\ 0^s \qquad \text{Decl. } +54^\circ\ 0'$$

For the coordinates of the projection of the zenith on the plate (expressed in parts of the focal length) we find

$$X = -0{\cdot}837 \qquad Y = -0{\cdot}056$$

and the diagrams give the following data for the conics :

Δx *conics.*

b =	8·85	$a/b = 1{\cdot}64$	$q+\phi = \cot^{-1}(0{\cdot}05)$
$\sqrt{3}\,.\,b$	15·3	Ellipses :	
$\sqrt{5}\,.\,b$	19·8	Corrections positive.	
$\sqrt{7}\,.\,b$	23·5		
$\sqrt{9}\,.\,b$	26·6		

Δy *conics.*

$b' = 18{\cdot}3 \quad a'/b' = \sqrt{-1} \times 1{\cdot}4 \quad q+\phi' = \tan^{-1}(0{\cdot}96)$

Hyperbolas :

Corrections positive.

And in the conjugate hyperbolas corrections negative.*

The resulting plate diagram is shown in figs. 2 and 3. The Δx and Δy diagrams have been kept separate for the sake of clearness on the small scale of the reproduction. For the same reason every tenth line only of the squared paper has been shown. In practice the two diagrams are drawn on the same centre, one of them in red ink. And there is no confusion unless the zenith distance is great and the circles very close together. The paper used is ruled with ten squares to the inch ; and one-tenth of an inch on the diagram corresponds to our unit, the thousandth of the focal length, or $3'{\cdot}438$ nearly.

The thick lined square represents the extreme limits of

* In the figure I have shown the conjugate hyperbolas wrong. Their vertices should come just on to the corners of the plate.

measurement on an astrographic plate. The minor axes of the conics are distinguished by the arrowhead. The stars for which

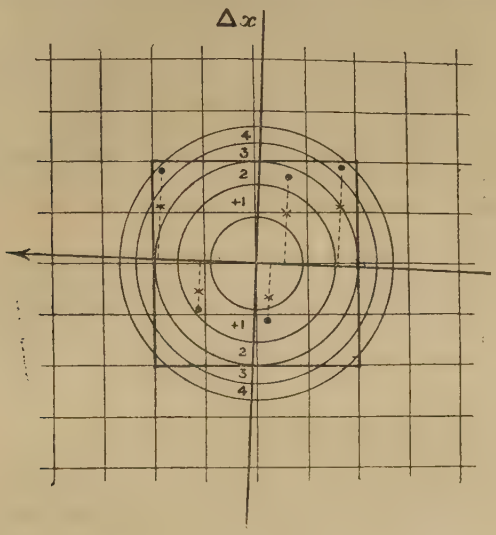


Fig. 2.

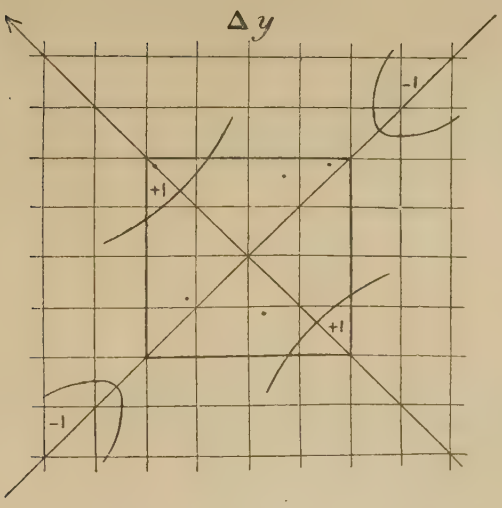


Fig. 3.

the corrections are wanted are plotted on the plate diagram, and the proportional compass is set to the ratio 1 : 1·64. The perpendiculars to the minor axis from the stars are thus divided in

the ratio 100 : 64 ; the points of division are shown by crosses ; and it follows from the way these points have been found that they are on the auxiliary circles of the ellipses of the series which pass respectively through the stars. They will therefore be inside or outside the auxiliary circles drawn on the diagram, according as their corresponding stars are inside or outside the ellipses which bound the zones of given correction.

The use of these diagrams is far more difficult to describe than it is to acquire. And in three cases out of four it is unnecessary to draw perpendiculars or to use the compasses at all ; it is obvious by inspection in which zone the "auxiliary circle point" will fall.

For instance, take the case of the five stars shown on the Δx diagram of the example. If we count from left to right it would have been obvious by inspection that the corrections for the first four stars are + 3, + 1, 0, + 1. For the fifth the auxiliary circle point comes very near the circle dividing zones 2 and 3 ; on drawing it we find that it is just inside the circle, and the correction is + 2.

As for the Δy corrections, that of the first is + 1, and of the others zero.

14. It is proper to remark here that occasionally—in perhaps 1 per cent. of cases—there will be a doubt whether the diagram has been drawn with sufficient accuracy to discriminate, we will say, between 2 and 3 ; the auxiliary point is practically on the dividing line. This means, of course, that the real correction is $2\frac{1}{2}$, and it is immaterial whether we call it 2 or 3. The diagram never introduces, therefore, any sensible inaccuracy.

15. I have recently used these diagrams for finding the second-order refraction corrections on a considerable series of *Eros* plates ; and it seems to me that they are a real working advantage. A rough estimate of the time required to correct a plate by their aid is :

Calculation of X and Y ... 5 min.

(this has to be done anyhow).

Drawing the plate diagrams ... 1—15 min.

(this depends entirely on the zenith distance. In many cases one finds immediately that there is no diagram wanted ; the first conic is outside the limits of the plate).

When the plate diagram is drawn the star corrections can be taken from it at the rate of four or five a minute at least.

Finally, a particular advantage of this method is that it saves all worry about whether second-order refraction terms are sensible or not, and all danger of neglecting them by inadvertence or otherwise. It is so easy to work off the diagrams for a whole set of plates that one finds it easier to draw the diagram

for every plate than to consider whether it will be necessary to do so in particular cases.

In the calculation of the tables from which the diagrams were drawn I have received valuable assistance from Miss Julia Bell and Miss Hilda Gibb, of Girton College, to whom my acknowledgments are due.

We are indebted to the Government Grant Fund of the Royal Society for a grant to secure their assistance in the reduction of the *Eros* photographs.

Cambridge Observatory: 1903 *January* 3.

Note on the Use of Mr. Aldis' Tables of the Function $\frac{1}{2}(\theta + \cos\theta)$ *in Determining the Elements of an Orbit.* By H. C. Plummer, M.A.

1. In the second part of his explanation* of the use of tables of $\frac{1}{2}(\theta + \cos\theta)$ Mr. Aldis has shown that they can be applied to the problem of finding the elements of an elliptic orbit when two focal distances, the angle between them, and the time occupied in passing from one position to the other are known. An inspection of the example which he has worked out makes it evident that the practical utility of the method may be greatly enhanced if a convenient way can be devised of finding preliminary approximations to the values of the quantities required. It is the object of this note to supply the want. A graphical method will be first described; but this is to be regarded chiefly as the basis of the analytical method of approximation which follows.

2. By the use of Lambert's theorem the problem is reduced to the solution for a, ϵ, and δ of this system of three equations:

$$\frac{kt}{a^{\frac{3}{2}}} = (\epsilon - \sin\epsilon) - (\delta - \sin\delta) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$1 - \cos\delta = \frac{r + r' - c}{2a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$1 - \cos\epsilon = \frac{r + r' + c}{2a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

The notation is that of Mr. Aldis' paper. When a is known the eccentricity e can be easily deduced. It will be convenient to write

$$\sigma_1^2 = r + r' + c\,; \quad \sigma_2^2 = r + r' - c.$$

Now if

$$x_1 = \delta - \sin\delta\,; \quad y_1 = 1 - \cos\delta$$

$$x_2 = \epsilon - \sin\epsilon\,; \quad y_2 = 1 - \cos\epsilon$$

* *Monthly Notices*, vol. lxii. p. 638.

x_1, y_1 and x_2, y_2 are the coordinates of points lying on the cycloid whose equations are

$$x = \theta - \sin\theta\,;\ y = 1 - \cos\theta$$

The coordinates are also connected by the relations

$$kt = a^{\frac{1}{3}}(x_2 - x_1)$$
$$2ay_1 = \sigma_2^2\,;\ 2ay_2 = \sigma_1^2$$

If a be eliminated we have

$$2\sqrt{2}kty_1^{\frac{1}{3}} = \sigma_2^3(x_2 - x_1)$$
$$2\sqrt{2}kty_2^{\frac{1}{3}} = \sigma_1^3(x_2 - x_1)$$

3. These equations have a geometrical interpretation. Let p and q (fig. 1) be the points (x_1, y_1) and (x_2, y_2) on the cycloid

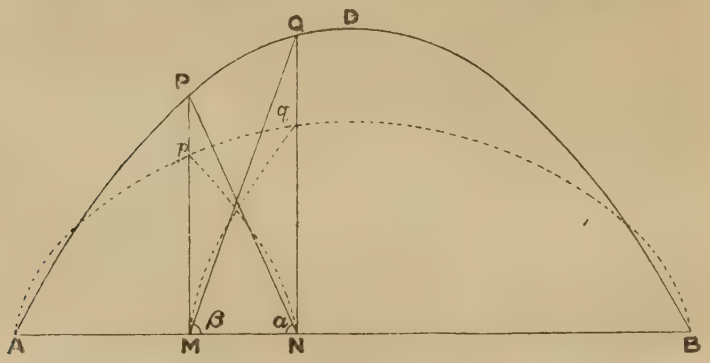


FIG. 1.

ACB. The meaning of the equations is that semi-cubical parabolas with known parameters and axes parallel to $x = 0$ will pass respectively through p and q when their vertices are at N and M, the feet of the ordinates at q and p. It then becomes evident that the figure will be greatly simplified by a transformation which corresponds to the substitution

$$x = X\,;\ y^{\frac{1}{3}} = Y$$

i.e. by raising each ordinate to the power $\frac{3}{2}$. The derived curve ADB (fig. 1) which replaces the cycloid has for its equations

$$X = \theta - \sin\theta\,;\ Y = (1 - \cos\theta)^{\frac{3}{2}} = 2\sqrt{2}\sin^3\tfrac{1}{2}\theta \quad \ldots \quad (4)$$

The semi-cubical parabolas are replaced by the straight lines PN, MQ, inclined to AB at angles α, β, such that

$$\tan\alpha = \sigma_2^3/2\sqrt{2}kt\,;\ \tan\beta = \sigma_1^3/2\sqrt{2}kt \quad \ldots \quad (5)$$

4. A graphical method of finding the required approximations is thus suggested. The curve ADB given by (4) must first be constructed on squared paper, and the base line AB may conveniently be graduated in θ. Then it is necessary to place the lines PN, QM at known inclinations to the base in such a manner that their intersections with the curve project (as in fig. 1) on to their intersections with the base. A convenient way of doing this is to draw the lines PN, QM on ground glass, with a third line in the direction of the base, and to move the figure so obtained over the paper until as good a fit as possible is obtained. Rough values of ϵ and δ can then be read off. But actual trial of the method in one or two cases has seemed to show that it is not susceptible of any great accuracy. This is in a great measure due to the fact that the given interval of time is in general small. The analytical method of approximation now to be described takes advantage of this fact, and will become the more effective as the graphical method becomes more impracticable.

5. In fig. 2, which corresponds to the essential part of fig. 1,

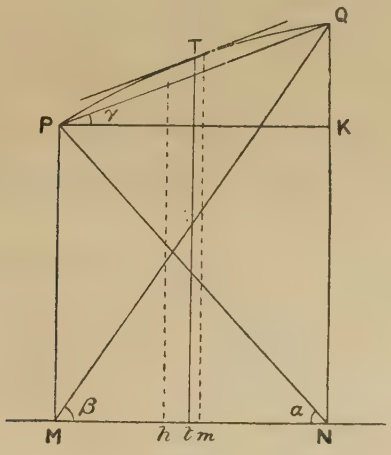


Fig. 2

let PK be drawn parallel to MN, and let the angle QPK $= \gamma$. This angle is easily determined, for

$$\tan\gamma = \tan\beta - \tan\alpha$$

Let T be the point of contact of that tangent to the curve which is parallel to PQ, and let $\theta = \tau$ at this point. It is easily found that

$$\tan\gamma = \frac{3}{\sqrt{2}}\cos\tfrac{1}{2}\tau$$

Hence by (5)

$$\cos \tfrac{1}{2}\tau = (\sigma_1^3 - \sigma_2^3)/6kt \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

a result which presents a curious analogy to Euler's theorem. In fact, the latter is hereby deduced from Lambert's theorem * if it is noticed that, in the limiting case of the parabola, $\delta = \epsilon = \tau = 0$.

6. It will be convenient to put

$$2\eta = \epsilon + \delta\,; \quad 2\zeta = \epsilon - \delta \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Now τ must clearly lie between δ and ϵ, and is indeed a good first aproximation to the value of η. Let ζ_1 be the corresponding value of ζ which makes (2) and (3) consistent. But these equations give

$$\sigma_1 \sin \tfrac{1}{2}\delta = \sigma_2 \sin \tfrac{1}{2}\epsilon$$

which can be written in the modified form

$$\tan \tfrac{1}{2}\zeta = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} \tan \tfrac{1}{2}\eta \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Hence

$$\tan \tfrac{1}{2}\zeta_1 = \frac{1-n}{1+n} \tan \tfrac{1}{2}\tau \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

where $n = \sigma_2/\sigma_1$. Thus τ and ζ_1 are found very easily by (6) and (9), and the corresponding approximations to δ and ϵ are deduced immediately.

7. It is now easy to see how the approximation can be improved by a simple step. Let (fig. 2) t be the foot of the ordinate through T, m the foot of the ordinate midway between PM and QN, and h the foot of the ordinate through the point for which $\theta = \frac{1}{2}(\epsilon + \delta)$. Let $y = \phi(x)$ be the equation of the curve and x and $x+h$ the abscissæ of P and Q. Then by a well-known theorem

$$\frac{\phi(x+h) - \phi(x)}{h} = \phi'(x + fh)$$

where

$$f = \tfrac{1}{2} + \tfrac{1}{24} \cdot \frac{\phi'''(x)}{\phi''(x)}\, h$$

if the expansion of f is not carried beyond the first power of x. Hence

$$tm = -\tfrac{1}{24} \cdot \frac{\phi'''(x)}{\phi''(x)}\, h^2$$

* Adams has given a simple proof that Euler's theorem can be deduced as a limiting case.—*Collected Scientific Papers*, vol. i. p. 413.

And if $x=\psi(\theta)$ and θ becomes $\theta+\Delta\theta$ at Q, then

$$km = \tfrac{1}{2}\psi(\theta)+\tfrac{1}{2}\psi(\theta+\Delta\theta)-\psi(\theta+\tfrac{1}{2}\Delta\theta)$$
$$= \tfrac{1}{8}\psi''(\theta)\,.\,\Delta\theta^2$$
$$\therefore ht = \left\{\tfrac{1}{8}\psi''(\theta)+\tfrac{1}{24}\,.\,\frac{\phi'''(x)}{\phi''(x)}\,[\psi'(\theta)]^2\right\}\Delta\theta^2$$
$$= +\tfrac{5}{48}\sin\theta\,.\,\Delta\theta^2$$

when the differentiations are performed. The correction to be applied to τ in order to give a second approximation to the value of η is therefore

$$-ht\div\psi'(\theta) = -\tfrac{5}{48}\cot\tfrac{1}{2}\theta\,.\,\Delta\theta^2$$

If, as above, the calculation is made in terms of the lower end of the chord, $\theta=\delta$; and starting from the upper end we should find $\theta=\epsilon$. It is therefore natural to adopt $\theta=\tau$. Also $\Delta\theta=2\zeta$. Hence it is inferred that if

$$\Delta\tau = -\tfrac{5}{12}\cot\tfrac{1}{2}\tau\,.\,\zeta^2 \quad \ldots\ \ldots\ \ldots\ (10)$$

then $\tau+\Delta\tau$ is the second approximation to the value of η, from which improved values of ϵ and δ can be deduced in the manner already explained. All the angles occurring in the foregoing formulæ and the constant k are supposed to be expressed in circular measure.

8. The result of practical trials is to show that the method is extremely effective. In the first instance the example which was taken by Mr. Aldis from the *Theoria Motus* was tried. This gave

$$\tau = 7^\circ\,38'\,21''{\cdot}6\,;\quad \zeta_1 = 3^\circ\,8'\,21''{\cdot}6\,;\quad \epsilon_1 = 81^\circ\,46'\,43''{\cdot}2$$
$$-\Delta\tau = \quad 5\ 14{\cdot}8\,;\quad \zeta_2 = 3\ 8\ 4{\cdot}0\,;\quad \epsilon_2 = 81\ 41\ 10{\cdot}8$$

Now Mr. Aldis finds for the correct value of ϵ, 81° 41′ 11″·5, and hence for log a, 0·4224385, while the value given by Gauss for log a is 0·4224389. The above value of ϵ_2 gives log a = 0·4224400. And if $\Delta\tau$ is recalculated by means of ζ_2 in the place of ζ_1, the correct value of ϵ is obtained. A second trial was made with an artificial example given by Dr. Moulton * in which the eccentricity was unusually large. In this case the value assigned to a was 2·65. The logarithms of three radii vectores are given only to six figures. The value found for ϵ_2 from the shorter period of thirty days gives $a=2{\cdot}650063$, which shows a fair agreement with the value actually assigned. But if the extreme places of the planet, with an interval of sixty-five days, are taken, the case is by no means so good. From the value found for ϵ_2, $a=2{\cdot}6519$. It is true that an improvement can be made by recalculating $\Delta\tau$ as mentioned above, but the

* *Astr. Journal*, No. 510.

method would lose the advantage of its simplicity thereby. It is better to treat ϵ_2 as an approximation which facilitates the use of Mr. Aldis' tables. Cases in which the interval of time is so long must occur very rarely, if at all, in the practical applications of the problem. But in such cases, and whenever the highest degree of accuracy is required, the value of the tables, as far as this problem is concerned, will consist in giving the final improvement, or at least the verification, to approximate solutions found by some preliminary method.

9. It is, however, desirable to examine the nature of the second approximation more closely. The tangent to the curve (4) at the point $\theta=\tau$ is parallel to the chord joining the points $\theta=\eta-\zeta$ and $\theta=\eta+\zeta$. Therefore

$$\tfrac{3}{4}\cos\tfrac{1}{2}\tau=\frac{\sin^3\tfrac{1}{2}(\eta+\zeta)-\sin^3\tfrac{1}{2}(\eta-\zeta)}{2\zeta-\sin(\eta+\zeta)+\sin(\eta-\zeta)}$$

$$\therefore 3\cos\tfrac{1}{2}\tau=\frac{3\cos\tfrac{1}{2}\eta\sin\tfrac{1}{2}\zeta-\cos\tfrac{3}{2}\eta\sin\tfrac{3}{2}\zeta}{\zeta-\cos\eta\sin\zeta}$$

$$\therefore \frac{3\cos\tfrac{1}{2}\tau}{2\cos\tfrac{1}{2}\eta}$$

$$=\frac{\tfrac{1}{2}\zeta(3c_1-3c_3)-\tfrac{1}{6.8}\zeta^3(3c_1-27c_3)+\tfrac{1}{120.32}\zeta^5(3c_1-243c_3)-\ldots}{\zeta(c_1-c_3)+\tfrac{1}{6}\zeta^3(c_1+c_3)-\tfrac{1}{120}\zeta^5(c_1+c_3)+\ldots}$$

where

$$c_1=\cos\tfrac{1}{2}\eta\,;\quad c_3=\cos\tfrac{3}{2}\eta$$

The expansion in ζ is clearly an even function, and if terms of the order ζ^6 be neglected

$$\frac{\cos\tfrac{1}{2}\tau}{\cos\tfrac{1}{2}\eta}=\frac{(c_1-c_3)-\tfrac{1}{6.4}\zeta^2(c_1-9c_3)+\tfrac{1}{120.16}\zeta^4(c_1-81c_3)}{(c_1-c_3)+\tfrac{1}{6}\zeta^2(c_1+c_3)-\tfrac{1}{120}\zeta^4(c_1+c_3)}$$

$$\therefore \frac{\cos\tfrac{1}{2}\eta-\cos\tfrac{1}{2}\tau}{\cos\tfrac{1}{2}\tau}=\frac{\tfrac{5}{24}\zeta^2(c_1-c_3)-\tfrac{1}{1920}\zeta^4(17c_1-65c_3)}{(c_1-c_3)-\tfrac{1}{24}\zeta^2(c_1-9c_3)}$$

$$=\tfrac{5}{24}\zeta^2-\tfrac{1}{5760}\cdot\frac{c_1+255c_3}{c_1-c_3}\cdot\zeta^4$$

$$=\tfrac{5}{24}\zeta^2+\tfrac{1}{5760}(191-64\cot^2\tfrac{1}{2}\eta)\zeta^4$$

10. In the coefficient of ζ^4, τ may be written for η. Then

$$\sec\tfrac{1}{2}\tau\,\Delta\cos\tfrac{1}{2}\tau=\tfrac{5}{24}\zeta^2+\tfrac{1}{5760}(191-64\cot^2\tfrac{1}{2}\tau)\zeta^4$$

where $\Delta\cos\frac{1}{2}\tau$ is the correction to be added to $\cos\frac{1}{2}\tau$ to obtain $\cos\frac{1}{2}\eta$, so that

$$\Delta\cos\tfrac{1}{2}\tau=-\tfrac{1}{2}\sin\tfrac{1}{2}\tau\,.\,\Delta\tau-\tfrac{1}{8}\cos\tfrac{1}{2}\tau(\Delta\tau)^2$$

Hence, to the degree of approximation adopted,

$$\Delta\tau=-\tfrac{5}{12}\cot\tfrac{1}{2}\tau\,.\,\zeta^2-\tfrac{1}{2880}\cot\tfrac{1}{2}\tau(191+61\cot^2\tfrac{1}{2}\tau)\zeta^4$$

which confirms the formula (10) previously obtained and gives a further development. Now since

$$4\tan^2\tfrac{1}{2}\tau=\zeta^2+\tfrac{1}{6}\zeta^4$$

it is possible to write

$$\Delta\tau=-\tfrac{5}{3}\cot\tfrac{1}{2}\tau\tan^2\tfrac{1}{2}\tau-\tfrac{1}{2880}\cot\tfrac{1}{2}\tau(61\cot^2\tfrac{1}{2}\tau-9)\zeta^4\ldots\quad(11)$$

Therefore if powers of ζ above the third be neglected it is more accurate to write (10) in the form

$$\Delta\tau=-\tfrac{5}{3}\cot\tfrac{1}{2}\tau\tan^2\tfrac{1}{2}\zeta$$

which again, by means of (9), may be expressed

$$\Delta\tau=-\tfrac{5}{3}\left(\frac{1-n}{1+n}\right)^2\tan\tfrac{1}{2}\tau\quad\ldots\quad\ldots\quad\ldots\quad(12)$$

Thus the second approximation to η may be derived directly from (6) and (12) before an approximation to ζ has been calculated. The work to this stage will therefore be very rapid, and it will be easy to judge from the value then found for ζ whether it is worth while to refine on the approximation by means of (11).

11. The representation of the consequences of Lambert's theorem by means of a curve related to the cycloid was suggested by the method I have previously described * for finding graphically an approximation to the solution of Kepler's equation by means of the trochoid. The application of Mr. Aldis' tables to the latter problem will be rendered more advantageous by using them as an auxiliary to some such approximate method as the one to which I refer. My description of that method was not, as it soon appeared, original. It had, for instance, been published by Dr. Rambaut † a few years earlier; and quite lately it has again been brought forward by B. Gonggrijp. ‡ But the principle of the method is as old as Wallis and Newton.

Note added 1903 *Jan.* 24.

The purpose of the foregoing note was, as explained in the beginning, merely to supply an approximate method which might serve to facilitate the use of Mr. Aldis' tables, and the examples mentioned in § 8 suffice to show that this object has been fully attained. But the method as further developed in §§ 9 and 10 is something more than a mere auxiliary to the tables. It provides an adequate and convenient means of solving Lambert's equation in cases of moderate difficulty without the aid of any special tables.

The essence of the method is contained in the equations (6),

* *Monthly Notices*, vol. lvi. (1896), p. 317.
† *Ibid.* vol. l. (1890), p. 301.
‡ *A. N.* (1901), No. 3720.

(8) and (11), by means of which the values of η and ζ are found by successive approximations. It is well to remark that

$$\zeta = \tfrac{1}{2}(E' - E)\,; \quad \cos\eta = e\cos\tfrac{1}{2}(E' + E)$$

where E, E′ are the eccentric anomalies of the planet ; and that

$$ap = rr' \sin^2\Delta \operatorname{cosec}^2\zeta$$

where 2Δ is the difference of the true anomalies. This explains the nature of the expansion (11), which might, if necessary, be developed further without serious difficulty. Its advantage consists in its rapid convergence for moderate values of ζ and in the fact that to the order ζ^5 it consists of only two terms, of which the second is very small. For actual calculation (11) may be written in the form

$$-\tfrac{1}{2}\Delta\tau = Z + FZ^2 \qquad \ldots \qquad \ldots \qquad (11')$$

where

$$Z = \tfrac{5}{6}\cot\tfrac{1}{2}\tau\tan^2\tfrac{1}{2}\zeta$$

$$F = \tfrac{1}{250}(61 - 9\tan^2\tfrac{1}{2}\tau)\cot\tfrac{1}{2}\tau$$

When log Z is found, Z can be taken from a table of logarithmic sines ; and the following table, which will apply to eccentricities of at least 0·5, gives log F in such a way that the term FZ^2 will be found in seconds of arc :—

Table of log F.

½τ.	log F.	Diff.	½τ.	log F.	Diff.	½τ.	log F.	Diff.
°			°			°		
30	4·9186	−192	40	4·7305	−190	50	4·5238	−240
31	·8994	190	41	·7115	193	51	·4998	250
32	·8804	189	2	·6922	195	52	·4748	262
33	·8615	187	43	·6727	198	53	·4486	276
34	·8428	187	44	·6529	202	54	·4210	291
35	·8241	186	45	·6327	206	55	·3919	310
36	·8055	187	46	·6121	212	56	·3609	332
37	·7868	186	47	·5909	217	57	·3277	358
38	·7682	188	48	·5692	223	58	·2919	390
39	4·7494	−189	49	4·5469	−231	59	·2529	−430
						60	4·2099	

The more complete formulæ have been applied to the example already mentioned in § 8, in which the interval of time is 65 days and the eccentricity 0·3. The result is quite satisfactory, and shows that the formulæ are as accurate as the data require. As the second interval between the observations does not enter into this part of the computation, and is therefore unlimited, this example is a far more severe test than when two intervals of 30 and 35 days are used. The additional term FZ^2 amounts

to $4''\cdot77$. It may be necessary, as it is in this case, to repeat the calculation by means of (8) and (11′) two or three times, but the process is in practice very simple and rapid. It is convenient to use logarithmic tables giving every second of arc (*e.g.* Bagay's), to keep the openings for $\frac{1}{2}\eta$, $\frac{1}{2}\zeta$, and $\frac{1}{2}\Delta\tau$ marked, and instead of repeating the whole of the figures (which in any case are very few) it is sufficient merely to write the *corrections* in parallel columns. In fact, when σ_1 and σ_2 are known it takes very little time to find η and ζ, and hence a, p and e. The method is summarised in the following scheme :—

Summary of Calculation.

(1) Calculate σ_1, σ_2 by means of

$$\sigma_1^2 = r+r'+c\ ;\ \sigma_2^2 = r+r'-c$$

(2) Calculate $n=\sigma_2/\sigma_1$, and hence $\log N = \log(1-n) - \log(1+n)$

(3) Find $\frac{1}{2}\tau$ by means of

$$\cos\tfrac{1}{2}\tau = (\sigma_1^3-\sigma_2^3)/6kt$$

(4) Find $\frac{1}{2}\Delta\tau$ from

$$\tfrac{1}{2}\Delta\tau = -\tfrac{5}{6}N^2\tan\tfrac{1}{2}\tau$$

(5) Hence $\frac{1}{2}\eta_2 = \frac{1}{2}(\tau+\Delta\tau)$, and ζ_2 can be found from

$$\tan\tfrac{1}{2}\zeta_2 = N\tan\tfrac{1}{2}\eta_2$$

(6) With ζ_2 as an approximate value of ζ find $\log Z$ from

$$Z = \tfrac{5}{6}\cot\tfrac{1}{2}\tau\tan^2\tfrac{1}{2}\zeta$$

and recalculate $\frac{1}{2}\Delta\tau$ by means of

$$-\tfrac{1}{2}\Delta\tau = Z+FZ^2$$

(7) If necessary repeat the operations (5) and (6) until the equations

$$-\tfrac{1}{2}(\eta-\tau) = -\tfrac{1}{2}\Delta\tau = Z+FZ^2$$
$$\tan\tfrac{1}{2}\zeta = N\tan\tfrac{1}{2}\eta$$

are both satisfied.

(8) When η and ζ are thus known with sufficient accuracy a is found from

$$a = \sigma_1^2/4\sin^2\tfrac{1}{2}(\eta+\zeta)$$

and p from

$$ap = rr'\sin^2\Delta\operatorname{cosec}^2\zeta$$

Finally the eccentricity is found from a and p.

University Observatory, Oxford :
1903 *Jan.* 8.

On a New and Accurate Method of Determining Time, Latitude, and Azimuth with a Theodolite. By W. E. Cooke, M.A., Government Astronomer, Western Australia.

The *principle* of this method is not new. It was, I believe, first advocated by Mr. S. C. Chandler, who designed an instrument which he called an "Almucantar," developed the mathematical theory, and made a series of very fine observations at the Harvard College Observatory. An instrument of that kind, however, does not form part of a surveyor's ordinary outfit, whilst a theodolite does ; and my present purpose is to show how the method may be applied to this universal instrument, and how by its means results of far greater accuracy than those usually obtained may be deduced. Briefly, the spirit level can be used instead of a mercury flotation in order to ensure constancy in zenith distance, and this brings the method within the power of every surveyor.

I shall first show it in connection with a 5-inch theodolite, and state the results of a few observations already made. Afterwards I propose to show that with a 12-inch instrument results can be obtained which, I believe, exceed in accuracy anything in field work hitherto published.

Five-inch Theodolite.

The observation for time and latitude consists in observing the time of transit of *Nautical Almanac* stars over the horizontal thread when the instrument is set so as to sweep a small circle in the sky parallel with the horizon, and at an altitude *about* equal to the observer's latitude. For azimuth it will be necessary, in addition, to take the time of transit across the vertical thread and to read the azimuth circle. An ordinary watch which possesses a seconds hand will do for timekeeper. The advantages may as well be enumerated here.

For time and latitude errors of vernier reading do not exist, as we require the circles for approximate setting purposes only. Extreme accuracy of construction and adjustment is unnecessary. We require only ordinary care in levelling. Error of collimation is absolutely immaterial. We do not even require that the bubbles shall be properly adjusted. All we require in the way of adjustment is that the vertical axis shall be reasonably vertical, so that the cross bubble on the altitude circle shall remain *fairly* steady as the instrument is swung round in azimuth, and great accuracy is not required even for this. As a matter of fact the instrument I used had been standing in a corner for months, and I simply took it out and levelled it up.

Yet, notwithstanding this apparent want of respect, results of considerable accuracy may be easily obtained. Those obtained in two evenings' trial will be stated almost immediately.

For azimuth I am afraid we must still depend upon the accuracy of the circles, and this immediately introduces errors which appear absurdly large compared with those for time and latitude. Incidentally it shows what an advance has been made by adopting this method. But even for azimuth it is confidently believed that the portion of error due to the star observation is greatly reduced, and that as the result of an evening's work the instrumental error is deduced with an accuracy far exceeding the possible setting. I may state here that I am not a practised observer with a theodolite, and possibly those who are may be able to read the circle itself with greater accuracy; but with a flickering candle-lamp I found it sometimes difficult to see any marked difference between two adjoining vernier divisions, and I think it is doubtful whether any surveyor would care to guarantee his accuracy within 1′ for each of a series of stars, some of the readings of which had to be made rather hurriedly. For azimuth, then, it can only be claimed that errors due to the star observation itself are probably reduced considerably, and that the observation can be conveniently made at the same time as the others, one setting being all that is required for each star, from which time, latitude, and azimuth are all deduced.

And now the results may be given for comparison with other methods, after which I shall show how the observations were made and reduced. I used a 5-inch theodolite, a candle lamp, and an ordinary watch. I had one vertical and one horizontal wire and no assistant. The watch did not tick either seconds, half-seconds, or any particular fraction of a second, and I had to get the time of transit the best way I could. I compared the watch during the evening with the standard sidereal clock, and thus I know its exact error. I observed on two evenings, with the following results :—

Time.

	Watch fast by direct comparison with sidereal clock.		Watch fast by observation of stars.	
	m	s	m	s
1902 Dec. 17 ...	1	27·1	1	26·9
18 ...	2	01·8	2	01·9

Latitude.

	°	′	″
Dec. 17 ...	31	57	10·9
18 ...	31	57	09·6

The real value of latitude is unknown to a fraction of a second, as the errors of division of the transit circle have not yet been determined; but it is, as nearly as I can obtain it, almost exactly midway between these two values. Such a remarkable agreement with a 5-inch instrument and common watch might be regarded as a mere coincidence were not its general accuracy determined by other considerations. It will, for instance, be found that the 12-inch gave quite unexpected agreement over

five nights. The principle upon which the observations are made produces confidence. And in particular the individual results show very satisfactory inter-agreement. I reproduce here those for the 18th, which are the better, as on the 17th I was new to the method and rather flurried at times.

Individual Results for each Pair, December 18.

Time.		Latitude.		
m	s	°	′	″
2	01·8	31	57	10·4
	1·5			9·0
	1·4			10·6
	2·3			8·4

The mean of the four watch errors does not agree exactly with that already given, but I had a ninth star unappropriated which was worked in.

I am, of course, willing to allow something for luck in such very close work ; but I think it would not be possible to obtain anything like such results by the methods at present adopted.

Definitions.

I wish to define a few terms first. Let us adopt Chandler's name for the circle of reference. The almucantar, or small circle parallel to the horizon which passes through the celestial pole, will be called "the colatitude circle." There are in an ordinary 5-inch theodolite two screws which move the telescope in altitude when clamped. One moves it without disturbing any bubble. Let us call this one X. The other, usually consisting of a pair of antagonistic screws, moves both the telescope and the bubble to which the altitude circle is clamped. Let us call this screw Y, and it is particularly important to note the difference. X moves the telescope only, Y moves both telescope and bubble. This bubble we shall speak of as "the bubble." It is, in my instrument, the largest bubble of any except the stride, and extends from edge to edge of the vertical circle. It is the essential feature of the whole scheme, and the more sensitive it is the better. In mine one division equals 20″.

Other Definitions.

ϕ = observer's latitude, considered always positive.

α = star's R.A.

δ = star's declination, N being + and S −.

t = star's hour angle, + if west, − east.

θ = sidereal time of transit across colatitude circle.

A = star's azimuth at transit across colatitude circle.

Practice.

1. Prepare a working list of stars beforehand. Any stars down to the fifth magnitude may be taken from the *Nautical Almanac*. Two series are required. Let us call these

(*a*) *Prime vertical stars*, or those which cross the colatitude circle within about 20° N. or S. of the prime vertical. For Perth I take 0° to 30° south declination.

(*b*) *Latitude stars*, or those remote from the pole, whose hour angle is not greater than $2\frac{1}{2}$ hours, *i.e.* whose polar distance is within a few degrees, say 10°, of $(180°-2\phi)$. Of course it must be *less* than this quantity in order that the star may cross the colatitude circle.

Compute the sidereal time of observation and azimuth approximately by the following formulæ :—

$$\theta = \alpha + t$$

$$\text{where } t = \begin{cases} \tan\phi \tan(45°-\frac{1}{2}\delta) \text{ for northern latitudes} \\ \tan\phi \tan(45°+\frac{1}{2}\delta) \text{ for southern latitudes} \end{cases}$$

$$\text{and} \qquad \tan\tfrac{1}{2}A = \phi \tan t$$

using the best available value of ϕ for this purpose.

2. Set up and level instrument ; in particular see that the level just mentioned is about the middle of its run. Set the telescope at an altitude $= \phi +$ refraction, using approximate values, of course the closer the better. Screw X may be used for this purpose, but after the first star is taken it must not be touched until the set is complete. A few minutes before the first star is expected, set for it in azimuth and clamp ; and now set "the bubble" quite accurately by means of screw Y. This must be done as carefully as possible. It is, in fact, the one essential adjustment, and a readjustment by screw Y must be made just before each star is taken. If when the star is seen in the field it appears that it will not cross the horizontal wire at the computed time the altitude may be altered by means of screw X ; but this applies to the first star only. The reason for the error will be an inaccurate telescope setting, or perhaps a large error in horizontal collimation. It may be adjusted approximately prior to the first transit, but afterwards must not be handled. It should perhaps be pointed out that this adjustment is purely a matter of convenience. The observations will give correct results even with a fairly large error of adjustment, but "the bubble" *must* be adjusted at some standard reading before each observation, the best being obviously the position of mid-level.

If the star appears to be moving in such a manner that it will not pass near the intersection of the wires the azimuth slow-motion screw may be used at any time.

3. Observe the times of transit across the horizontal and vertical wires, moving the azimuth screw so that the star crosses near, but not quite at, the centre. Do not touch the azimuth screw after transit of the vertical wire, or if you do, move it so that the star has to cross the wire again and take the second transit instead of the first. Then read the azimuth circle.

If only time and latitude are required take the horizontal transit only, and do not trouble about the azimuth circle.

Computation.

By means of the bubble we have observed the transit of a number of stars over the horizontal wire at some one definite zenith distance, *i.e.* over a definite almucantar, the accuracy of which depends upon the accuracy with which we have adjusted the bubble each time. We are supposed to have observed a few prime vertical and a few latitude stars, both east and west of the meridian, and if the number observed is equally divided between east and west so much the better.

From the prime vertical stars we shall find first the quantity Z, whereby the circle actually traced by the instrument differs from the true colatitude circle, and secondly the watch error.

From the latitude stars combined with Z, just obtained, we shall compute our latitude.

From any of the stars we shall compute the instrumental error in azimuth.

Our method requires that the latitude should be approximately known. If it is known with fair accuracy we may go straight ahead, but if not it will be as well to make a preliminary computation, taking one pair of prime vertical stars for Z and one pair for latitude. This will give us a result quite accurate enough for use.

Computation for Time.

With this value of latitude compute rigorously for each star values of θ and A by the formulæ already given, but use now seven-figure logs and take angles out to seconds at least. Compute also with four-figure logs

$Z = \operatorname{cosec} t \sec \delta$. Z is + for west and − for east stars in all latitudes. This quantity is required for every star.

$L = \frac{2}{15 \sin 2\phi} \cot t$. Same sign as Z. Required for latitude stars only.

If τ represents the observed time of transit reduced to sidereal time, form the quantity $(\theta+\tau)$ for each star. Distinguish eastern observations with an accent. Thus $(\theta-\tau)$ for west and $(\theta'-\tau')$ for east, &c. If the watch is fast this quantity will probably be negative, and if slow positive. Arrange the prime

vertical stars in two columns, west and east stars, and opposite each star place its Z and $(\theta-\tau)$. Take the mean of each of the four columns. We shall thus have mean values of Z, $(\theta-\tau)$, Z′, and $(\theta'-\tau')$, Z and Z′ being of opposite signs.

Then $$z = \frac{(\theta-\tau)-(\theta'-\tau')}{Z-Z'}$$

The watch error is practically the mean between $(\theta-\tau)$ and $(\theta'-\tau')$, but more acurately it equals

$$(\theta-\tau)-Zz = (\theta'-\tau')-Z'z$$

Be careful about the sign of z, as the correction to latitude depends upon this.

Each pair of stars may be treated separately as above, if it be required to see how they agree amongst themselves.

Computation for Latitude.

Enter the latitude stars in two columns, east and west, and opposite each star write its Z, $\theta-\tau$, and L.

Compute $$a = (Z-Z')z$$
$$b = (\theta-\tau)-(\theta'-T')$$
$$c = L-L'$$

Then the correction to the latitude, or $\Delta\phi$, equals

$$\Delta\phi = \frac{b-a}{c}$$

and will be in seconds of arc. The sign obtained will be correct for either hemisphere, provided ϕ has been considered a positive quantity throughout; *i.e.* if $(b-a)$ is negative *subtract* $\Delta\phi$ numerically from the assumed value, and *vice versa*.

The computation may be applied to each pair separately, or to the mean of the east and mean of the west stars.

Computation for Azimuth.

Reduce the observed to sidereal time, apply the watch error, just obtained, and subtract this from the computed value of θ. Call this dt, and express it in seconds of time.

Compute $dA = 15 \sin A \cot t \,.\, dt$, using 4-figure logs, and apply this, which will be in seconds of arc, as a correction to the observed circle reading. The result will be instrumental azimuth, and the difference between it and A will be $= a$, the instrumental error in azimuth.

Observation with a 12-inch Theodolite.

The principle is the same as with a 5-inch. The modifications are :

1. The bubble is much larger. It is attached loosely, so as to revolve round the horizontal axis of the theodolite. It is set approximately level after the telescope-pointing in altitude has been made, and is then clamped firmly to the telescope. It is not adjusted before each observation. In fact, no alteration in altitude is made during the evening, but the bubble is read immediately after each transit, and a correction is made to reduce all observations to some definite level reading, arbitrarily chosen. One division is equal to $1''{\cdot}2$.

2. Instead of one horizontal and one vertical wire there are, in my instrument, three horizontal and five vertical ones. I make a point of obtaining transit over each of the three horizontal and as many as possible of the vertical wires.

3. The mean of the vertical wires is not necessarily in the line of collimation, but for azimuth a micrometer is used and the reading for collimation obtained. I have tried several methods, but none of them are perfectly satisfactory, in my opinion. This, however, is a trouble common to any method for obtaining azimuth, and one of the great charms of this new method of determining time and latitude is that collimation lines and circle readings are things of the past. For azimuth, however, all transits must be reduced to the vertical line of collimation.

4. A chronometer, or chronograph if available, is now used, and the results are sufficiently accurate to require a correction for diurnal aberration.

Computation.

Instead of repeating the whole method of computation I shall confine myself to indicating the change caused by the variation in the instrument.

1. Correction for bubble-reading. The value of one division of the scale must be known. Some definite bubble-reading is to be arbitrarily selected, preferably nearly the same as the mean of all the readings taken, and the difference between this standard and the observed reading, expressed in arc seconds, must be multiplied by

$$\frac{dt}{d\zeta} = \frac{1}{15 \cos\phi \sin A}$$

The result, which will be in seconds of time, must then be applied as a correction to the observed mean time of transit over the horizontal wires.

No rules for signs can be given, as these will depend upon the method of graduating, &c., and may be different in different instruments. The sign for west stars will be opposite that for east.

2. Reduction to mean of wires or line of collimation. For this purpose the interval between the mean of observed times and (*a*) the mean of wires, for horizontal, (*b*) the line of collimation, for vertical wires, must be known. To save trouble I made a point of always obtaining the three horizontal transits, and simply took the mean of the three. But this may not always be practicable, so the following formulæ are given :

Let f = angular interval of mean of observed from the ideal line to which all stars are to be referred.

F = correction to be applied.

Then for horizontal transits

$$F = Zf + \tfrac{1}{2}^{s}f^{2} \sin 1'' \cdot Z^{2} \cos t$$

and for the prime vertical set of stars only the first term need be used.

For transits over vertical threads

$$F = f \sec \phi$$

3. The correction for diurnal aberration is

$$\kappa = 0^{s}\cdot 0207 \sin \phi$$

This is to be *added* to the computed clock-error, or *subtracted* from the observed time of transit.

4. A correction for rate of chronometer should be applied to each transit, to reduce to some definite sidereal hour.

Results.

Stars were observed on five evenings, time being taken by the eye and ear method with a sidereal chronometer beating half-seconds. The chronometer was compared before and after the evening's work with the standard sidereal clock. The theodolite was placed upon a small brick pillar in the meridian of the transit circle, and the figures below show the differences between the errors of the chronometer deduced from comparison with a sidereal clock and those by direct observation with the theodolite. The error of the sidereal clock was obtained by means of transits observed with the transit circle.

	Time. Diff. τc — theodolite.	Latitude.
	s	° ′ ″
1902 Dec. 8	−0·03	31 57 10·3
9	+0·01	10·1
10	−0·07	10·5
12	+0·02	10·2
13	0·00	10·2

A sufficient number of observations have not been made to test azimuth results, but such as they are they appear satisfactory. The only rough and ready method that occurred to me was to take the instrumental error in azimuth for each night as deduced from prime vertical and latitude stars respectively. Only two sets are available, as it did not occur to me to try this method for azimuth until the 12th, and even then I adopted different tactics for the two sets. For the prime vertical stars I kept the star bisected on the middle vertical wire with the slow-motion screw in azimuth until it reached a fixed horizontal wire, and took no time transits at all, reducing by a different method. I did not like this way, however, as well as the time transit, and only used it on that one occasion ; but it is satisfactory to note the agreement between the two.

		P. Vert. Stars.	Latitude Stars.
Dec. 12	Inst. error in az.	+10″·2	+10″·7
13	„ „ „	+ 4·8	+ 4·8

Of course the change between the 12th and 13th is of no account. That was almost certainly a real change.

In conclusion I must express my great obligation to S. C. Chandler for his work on the Almucantar published as vol. xvii. of the *Annals of Harvard College Observatory*, to which I refer all those who wish to see more of the mathematics and results of this beautiful method.

On the Sun's Stellar Magnitude and the Parallax of Binary Stars. By J. E. Gore, M.R.I.A.

The Sun's stellar magnitude, or the number which represents the Sun's brightness as seen from the Earth on the same scale as the " magnitudes " of the stars are represented, has been variously estimated at numbers ranging from $-25{\cdot}5$ to $-27{\cdot}0$. The following method of computing its value from the apparent brightness of binary stars, whose orbits are well determined and whose spectra resembles that of the Sun, has not, so far as I am aware, been previously published.

Let P = period of binary star in years

a = semi-axis major of orbit in seconds of arc

p = parallax of binary

Then we have

$$\text{Mass of binary system} = \frac{a^3}{p^3 \mathrm{P}^2} \text{ (Sun's mass = 1)}$$

Now if we assume the binary pair to have the same density and surface brilliancy as the Sun, we have surface of single body equivalent in mass to the binary pair

$$= \left(\frac{a^3}{p^3 \mathrm{P}^2}\right)^{\frac{2}{3}} = \frac{a^2}{p^2 \mathrm{P}^{\frac{4}{3}}}$$

Then, as the relative brightness is proportional to the illuminated surface, we have

$$\text{Brightness of Sun} : \text{Brightness of binary} :: 1 : \frac{a^2}{p^2 \mathrm{P}^{\frac{4}{3}}}$$

and

$$\therefore \text{Brightness of Sun} = \frac{p^2 \mathrm{P}^{\frac{4}{3}}}{a^2} \times \text{Brightness of binary} = \mathrm{B} \text{ (say).}$$

Now if m = photometric magnitude of binary, we have

$$m - \frac{\log \mathrm{B}}{0{\cdot}4} = \text{magnitude of Sun at distance of binary} = \mathrm{M}$$

(0·4 = log "light ratio" = log 2·512)

or
$$m - \left\{ \frac{2 \log p + \frac{4}{3} \log \mathrm{P} - 2 \log a}{0{\cdot}4} \right\} = \mathrm{M}$$

or
$$m - 5 \log p - \tfrac{10}{3} \log \mathrm{P} + 5 \log a = \mathrm{M} \quad \ldots \quad \ldots \quad (1)$$

And we have also Sun's magnitude at distance of binary (if S = Sun's stellar magnitude)

$$\mathrm{M} = \frac{\log \left(\frac{206265}{p}\right)^2}{0{\cdot}4} + \mathrm{S} \text{ (S being negative)}$$

$$= 5 \log 206265 - 5 \log p + \mathrm{S}$$

or
$$\mathrm{M} = 26{\cdot}57 - 5 \log p + \mathrm{S} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Equating (1) and (2), we have

$$m - 5 \log p - \tfrac{10}{3} \log \mathrm{P} + 5 \log a = 26{\cdot}57 - 5 \log p + \mathrm{S}$$

whence
$$\mathrm{S} = m - 26{\cdot}57 - \tfrac{10}{3} \log \mathrm{P} + 5 \log a \quad \ldots \quad (3)$$

This equation depends only on m, the photometric magnitude of the binary star, P its period, and a the semi-axis major of the computed orbit, and is independent of the star's parallax or distance from the Earth.

As, however, the combined surface of a pair of spheres is greater than that of a single sphere having the same mass, equation (3) will require a small correction. The amount of this correction is easily found. For let R = radius of single sphere, and r and nr = radii of two spheres having a combined mass

equal to the single sphere. Then surface of single sphere is proportional to R^2, and combined surface of the two spheres is proportional to $n^2+n^2r^2 = (n^2+1)r^2$; and we have also

$$R^3 = n^3r^3+r^3 = (n^3+1)r^3$$

and $$\therefore R^2 = (n^3+1)^{\frac{2}{3}} r^2$$

and ratio of combined surfaces of the two spheres to surface of single sphere is

$$(n^2+1)r^2 : R^2$$

or $$(n^2+1)r^2 : (n^3+1)^{\frac{2}{3}} r^2$$

or $$n^2+1 : (n^3+1)^{\frac{2}{3}}$$

If the components of the binary star are equal in size the ratio is $2 : 2\frac{2}{3}$, or $2\frac{1}{3} : 1$, or $1{\cdot}26 : 1$.

If $n = 2$, the ratio is $5 : 9^{\frac{2}{3}}$, or $1{\cdot}15 : 1$. This corresponds to a difference of about $1\frac{1}{2}$ magnitude between the components.

If $n = 4$, the ratio is $17 : 16{\cdot}166$. This corresponds to a difference of about 3 magnitudes between the components.

The correction is then made by adding the logarithm of this ratio divided by $0{\cdot}4$ to equation (3). For the extreme case, when the components are equal in size, the equation becomes

$$S = m-26{\cdot}57-\tfrac{10}{3}\log P+5\log a+\frac{\log 1{\cdot}26}{0{\cdot}4}$$

or $$S = m-26{\cdot}57-\tfrac{10}{3}\log P+5\log a+0{\cdot}25$$

or $$S = m-26{\cdot}32-\tfrac{10}{3}\log P+5\log a \quad \ldots \quad \ldots \quad (4)$$

Let us now apply the formula to some examples of binary stars which have a spectrum closely resembling the solar spectrum, and of which the density and surface brilliancy are probably nearly the same as the Sun's.

α Centauri. $P = 81{\cdot}1$ years, $a = 17''{\cdot}7$ (See). Photometric magnitude $= 0{\cdot}20$ (mags. of components $0{\cdot}50$ and $1{\cdot}75$, Pickering). Spectrum of solar type. Here we have

$$\begin{aligned} S &= 0{\cdot}20-26{\cdot}57-\tfrac{10}{3}\log 81{\cdot}1+5\log 17{\cdot}7 \\ &= 0{\cdot}20-26{\cdot}57-6{\cdot}3634+6{\cdot}2398 \\ &= -26{\cdot}49 \end{aligned}$$

In this case the difference between the components being $1{\cdot}25$ magnitude, the correction comes out $+0{\cdot}18$, and hence

$$S = -26{\cdot}49+0{\cdot}18 = -26{\cdot}31$$

ξ Ursæ Maioris. $P = 60$ years, $a = 2''{\cdot}508$ (See). Mag.

= 3·93 (components 4, $4\frac{1}{2}$). Spectrum solar type (G. Pickering). Here we have

$$\begin{aligned} S &= 3{\cdot}93 - 26{\cdot}57 - \tfrac{10}{3}\log 60 + 5\log 2{\cdot}508 + 0{\cdot}23 \\ &= 3{\cdot}93 - 26{\cdot}57 - 5{\cdot}927 + 1{\cdot}996 + 0{\cdot}23 \\ &= -26{\cdot}34 \end{aligned}$$

η Cassiopeiæ. P = 500 years, $a = 11''{\cdot}4$ (Comstock). Mag. = 3·63 (components 4 and 7·6). Spectrum second type, F 8 G (Pickering).

Here we have

$$\begin{aligned} S &= 3{\cdot}63 - 26{\cdot}57 - \tfrac{10}{3}\log 500 + 5\log 11{\cdot}4 \\ &= 3{\cdot}63 - 26{\cdot}57 - 8{\cdot}99 + 5{\cdot}28 \\ &= -26\ 65 \end{aligned}$$

In this case, as there is a difference of 3·6 magnitudes between the components, the correction is very small and may be neglected.

As the parallax of *α Centauri* is known (0″·75), we can compute the Sun's stellar magnitude from it in another way. The mass of the brighter component is about equal to the Sun's mass, and the spectrum very similar (G. Pickering); we can therefore fairly compare the two bodies.

Let p = parallax of the star.

Then $\frac{206265}{p} \times 93000000$ = distance of star.

Let θ = apparent diameter of the star in seconds of arc.

Then $\theta : 206265 :: 866000 : \text{distance}$

(866000 = Sun's diameter in miles)

and $\therefore \quad \theta = \dfrac{206265 \times 866000}{\text{distance}}$

$$= \frac{206265 \times 866000}{\frac{206265}{p} \times 93000000}$$

or $$\theta = \frac{866000p}{93000000}$$

$$= \frac{p}{107} \text{ or } = 0''{\cdot}009345p$$

For *α Centauri* we have

$$\theta = 0{\cdot}009345 \times 0{\cdot}75 = 0''{\cdot}007$$

and

$$\text{Brightness of sun} = \frac{(1920'')^2}{(0''{\cdot}007)^2} = \frac{3686400}{0{\cdot}000049} = 75232650000$$

or the Sun is 75,232,650,000 times brighter than the brighter component of *a Centauri*.

Hence if the Sun and the brighter component of *a Centauri* have the same intrinsic brightness of surface and the same density, we have

$$(2{\cdot}512)^n = 75232650000$$

and

$$n = 27{\cdot}2$$

or the Sun is 27·2 magnitudes brighter than the brighter component of *a Centauri*. And as the photometric magnitude of the star is 0·50, we have

$$\text{Sun's stellar mag.} = (0{\cdot}50 - 27{\cdot}2) = -26{\cdot}70$$

The mean of the four results found above is

$$S = -26{\cdot}50$$

Substituting this value of S in equation (2), we have

$$\begin{aligned} M &= 26{\cdot}57 - 5\log p - 26{\cdot}50 \\ &= -5\log p + 0{\cdot}07 \\ &= 5\log\left(\frac{1}{p}\right) + 0{\cdot}07 \quad \ldots \quad \ldots \quad \ldots \quad (5) \end{aligned}$$

a simple formula for obtaining the Sun's magnitude if removed to the distance of a star whose parallax is p.

In the case of a spectroscopic binary star having a solar spectrum, if we assume that the plane of the orbit passes through the Earth, or nearly so, we can find from the observed "radial velocity" the dimensions of the system in miles, and the combined mass of the components. From these data we can compute the probable parallax of the star in the following way:

Let μ = mass of spectroscopic binary

and m = its photometric magnitude.

Then from equation (5) we have (neglecting the small constant)

$$\left(5\log\frac{1}{p} - m\right) \times 0{\cdot}4 = \log \mu^{\frac{2}{3}}$$

or,

$$2\log\frac{1}{p} - 0{\cdot}4m = \frac{2}{3}\log\mu$$

and

$$\therefore \log\frac{1}{p} = \frac{1}{3}\log\mu + 0{\cdot}2m \quad \ldots \quad \ldots \quad (6)$$

Or, to obtain the parallax directly from the period and com-

puted semi-axis major in miles $= d$, we have, if R = radius of Earth's orbit,

$$\mu = \frac{\left(\frac{d}{R}\right)^3}{P^2}$$

and $$\therefore \log \mu = 3 \log d - 3 \log R - 2 \log P$$

Substituting this in equation (6), we obtain

$$\log p = -\tfrac{1}{3}(3 \log d - 3 \log R - 2 \log P) - 0{\cdot}2m$$

or
$$\begin{aligned} \log p &= \tfrac{2}{3} \log P - 0{\cdot}2m + \log R - \log d \\ &= \tfrac{2}{3} \log P - 0{\cdot}2m + \log (92800000) - \log d \\ &= \tfrac{2}{3} \log P - 0{\cdot}2m + [7{\cdot}9675480] - \log d \qquad \ldots \quad (7) \end{aligned}$$

Take the case of η *Pegasi*, which is a spectroscopic binary with a spectrum of the solar type (G. Pickering). The magnitude is 3·20. The spectroscopic observations show that the period is 818 days, and the semi-axis major of the orbit about 200,000,000 miles. This gives a mass about twice the Sun's mass. Hence from equation (6) we have

$$\begin{aligned} \log \frac{1}{p} &= \frac{1}{3} \log 2 + 0{\cdot}2 \times 3{\cdot}20 \\ &= 0{\cdot}10034 + 0{\cdot}64 \\ &= 0{\cdot}74034 \end{aligned}$$

$$\therefore \frac{1}{p} = 5{\cdot}5, \text{ and } p = \frac{1}{5{\cdot}5} = 0''{\cdot}182$$

From equation (7) we have

$$\begin{aligned} \log p &= \tfrac{2}{3} \log \tfrac{818}{365} + 7{\cdot}9675480 - \log 200000000 - 0{\cdot}64 \\ &= 0{\cdot}2334986 + 7{\cdot}9675480 - 8{\cdot}3010300 - 0{\cdot}64 \end{aligned}$$

or $$\log p = \bar{1}{\cdot}2600166$$

and $$p = 0''{\cdot}182, \text{ as before.}$$

With this parallax I find, from equation (5), that the Sun would be reduced to a star of 3·77 magnitude if removed to the distance of η *Pegasi*. This makes the star 0·57 mag. brighter than the Sun, and its mass about 2·2 times the Sun's mass; a result which agrees closely with the mass derived from the spectroscopic observations.

In the case of spectroscopic binaries which have a spectrum of the *Sirian* type their light is not comparable with that of the Sun, and the above formulæ cannot be used. But we can compare them with *Sirius*, of which the parallax and mass are known, and thus deduce a probable parallax.

Taking the parallax of *Sirius* at 0″·38 and the mass of the bright star, as deduced from See's orbit,* at 2·36, let p = parallax of binary, and μ its mass computed from the spectroscopic observations. Then we have ratio of light of *Sirius* to light of spectroscopic binary

$$= \left(\frac{0{\cdot}38^2}{p^2}\right) \times \left(\frac{2\,3\,6}{\mu}\right)^{\frac{2}{3}}$$

and if m = magnitude of spectroscopic binary, we have, taking the magnitude of *Sirius* = − 1·62

$$\log \text{ ratio of light} = \{m - (-1{\cdot}62)\} \times 0{\cdot}4 = 0{\cdot}4m + 0{\cdot}648$$

Hence

$$\log \left\{ \left(\frac{0{\cdot}38}{p}\right)^2 \times \left(\frac{2{\cdot}36}{\mu}\right)^{\frac{2}{3}} \right\} = 0{\cdot}4m + 0{\cdot}648$$

or

$$2 \log 0{\cdot}38 + \tfrac{2}{3} \log 2{\cdot}36 - 2 \log p - \tfrac{2}{3} \log \mu = 0{\cdot}4 \text{ m} + 0{\cdot}648$$

or

$$\bar{1}{\cdot}1595 + 0{\cdot}2486 - 0{\cdot}648 - 2 \log p - \tfrac{2}{3} \log \mu = 0{\cdot}4m$$

whence

$$\log p = \bar{1}{\cdot}380 - \tfrac{1}{3} \log \mu - 0{\cdot}2\, m \quad \ldots \quad \ldots \quad (8)$$

Let us take some examples of the application of this formula.

Algol. Vogel finds the mass of the bright component of this well-known variable star to be $\frac{4}{9}$th of the Sun's mass = 0·4444 × Sun.

Its photometric magnitude at normal brightness as measured at Harvard is 2·31. Hence we have

$$\begin{aligned} \log p &= \bar{1}{\cdot}380 - \tfrac{1}{3} \log 0{\cdot}4444 - 0{\cdot}2 \times 2{\cdot}31 \\ &= \bar{1}{\cdot}380 - \bar{1}{\cdot}8826 - 0{\cdot}462 \\ &= \bar{1}{\cdot}035 \end{aligned}$$

and $p = 0''{\cdot}1082$

Chandler computed a parallax of 0″·07.

Spica. Mag. 1·09. Vogel found from the spectroscopic observations a combined mass of 2·6 × Sun's mass. Assuming the bright and dark components to be equal in mass, we have mass of bright component = 1·3, and hence :

$$\begin{aligned} \log p &= \bar{1}{\cdot}380 - 0{\cdot}0379 - 0{\cdot}218 \\ &= \bar{1}{\cdot}1241, \text{ and } p = 0''{\cdot}133 \end{aligned}$$

As the star is not an *Algol* variable, the plane of the orbit

* *Researches on the Evolution of the Stellar Systems*, vol. i. p. 85.

does not probably pass through the Earth. Hence the mass is probably larger, and the parallax smaller, than that found above. No parallax has been found for this star.

β Aurigæ. Mag. 1·98. Computed mass = 5·5 × Sun's mass. In this case both components are bright, and Pickering says: "Both components appear to be equal, or nearly equal, in brightness and to have similar spectra (Spectrum A). We must therefore deduct from the constant in equation (8), $\frac{1}{2}$ log (1·26), or 0·05, and the formula becomes

$$\log p = \bar{1}\cdot330 - \tfrac{1}{3}\log \mu - 0\cdot2m$$

$$\text{or } \log p = \bar{1}\cdot330 - \tfrac{1}{3}\log 5\cdot5 - 0\cdot2 \times 1\cdot98$$
$$= \bar{1}\cdot330 - 0\cdot2467 - 0\cdot396$$
$$= \bar{2}\cdot6873, \text{ and } \therefore p = 0''\cdot04868$$

The late Professor Pritchard found by means of photography a parallax of 0″·065 and 0″·059.

ζ Ursæ Majoris. Mag. 2·18. (Both components bright.) Period according to Vogel and Eberhard 20·6 days, and mass $= \frac{4 \times \text{Sun's mass}}{\sin^3 i}$.

If we suppose $i = 90°$, $\sin^3 i = 1$, and the mass will be 4 × Sun's mass. Supposing the components to be equal in brightness, we can use the same formula as for *β Aurigæ*, and

$$\log p = \bar{1}\cdot330 - \tfrac{1}{3}\log 4 - 0\cdot2 \times 2\cdot18$$

This gives $\log p = \bar{2}\cdot6933$, and $\therefore p = 0''\cdot049$

Höffler found a parallax of 0″·0165, but a parallax of 0″·045 was found by Klinkerfues.

Castor. Mag. 1·61 (components about 1·98 and 2·98). The brighter component of this well-known binary star has been found to be a spectroscopic binary, the companion being a dark body like *Algol*. Period 2·98 days. Orbital velocity = 20·7 miles a second. If we suppose, as in the case of *Algol*, that the bright component has twice the mass of the dark companion, I find that $a = 2{,}550{,}000$ miles $(= 1{,}700{,}000 \times \frac{3}{2})$, and the mass = 0·3113 × Sun's mass. Hence mass of bright component = 0·3113 × $\frac{2}{3}$ = 0·2075, and we have

$$\log p = \bar{1}\cdot380 - \tfrac{1}{3}\log(0\cdot2075) - 0\cdot2 \times 1\cdot98$$
$$= \bar{1}\cdot2117 \text{ and } \therefore p = 0''\cdot1628$$

From heliometer measures of *Castor* made by Johnson at Oxford in 1854–55 he found a parallax of 0″·198.

The parallaxes found above for spectroscopic binaries have been computed on the assumption that the plane of the orbit passes through the Sun, or nearly so. If the orbital plane is inclined to the line of sight—as it probably is in most cases—

the mass of the system will be greater and the parallax consequently less. If the data used are correct, the computed values are therefore a maximum.

With reference to stars having spectra between the second and third types, the star λ *Andromedæ* is, I find, an interesting case. The spectroscopic observations show a period of about 19·2 days and an orbital velocity of about 5·6 miles a second. From this I find (supposing the orbit plane to be in the line of sight) a mass of only 0·0119 × Sun's mass. As the star's spectrum is K, according to Pickering it is not comparable with the Sun, and the formulæ given above cannot be used in this case; but as its photometric magnitude is 4·14, the very small mass would suggest that the star is comparatively near the Earth. If we suppose the inclination of the orbit to be 30°, the mass would be increased eight times; but even then the mass would be less than one-tenth of the Sun's mass.

Dublin: 1903 *January* 4.

New Double Stars detected with the $17\frac{1}{4}$*-inch reflector during the year* 1902. By T. E. Espin.

The following list contains new double stars found during 1902. The weather has been continuously bad throughout the year, and even when the sky has been clear the definition has generally been very poor. Many of the *comites* are far too faint to measure satisfactorily.

No.	B.D.	R.A. 1880. h m	Decl. ° ′	P. °	D. ″	Night.	Mags.		Note.
113	66°6	0 2·6	+66 37	122·6	6·8	1	8·5	11	...
114	66·7	3·1	66 29	161·6	5·0	3	8·7	11·2	...
115	61·50	16·3	61 34	82·4	9·9	1	8·0	10	...
116	54·87	24·6	54 59	255·9	7·7	1	8·9	8·9	...
117	54·106	27·6	55 3	54·4	3·0	1	9·0	11	...
118	63·111	47·7	63 43	241·3	2·6	2	8·6	8·7	...
119	53·271	1 10·3	54 19	115·1	5·1	1	8·2	10·5	...
120	53·576	2 41·1	53 26	70·3	3·9	1	8·7	12·5	...
121	57·729	3 24·1	57 51	325·5	6·9	2	8·0	13·5	...
122	61·666	55·3	61 50	248·7	5·0	2	8·6	10·5	...
123	44·2120	11 37·4	44 51	203·8	7·4	1	9·1	9·3	AB.
				275·5	42·9	1		9·2	AC.
124	42·2287	12 10·2	42 34	135 ±	5 ±	1	9·0	12·5	Too faint.
125	42·2370	59·9	42 19	119·1	2·4	2	8·0	10·6	...

No.	B.D.	R.A. 1880. h m	Decl.	P.	D.	Night.	Mags.		Note.
	°		° ′	°	″				
126	63·1446	18 38·5	+63 41	21·9	4·9	3	11	12	BC faint.
				53·5	73·1	3		8·0	AB, 14 mag. between.
127	62·1649	46·2	62 46	135·7	4·7	1	9·5	9·5	...
128	46·2659	19 13·4	46 58	281·6	4·7	2	8·4	11·5	...
129	53·2264	30·3	53 38		2·9	2	9·2	10·0	Note.
130	60·2017	45·8	+60 51	237·7	2·9	3	9·5	9·7	...
131	53·2323	52·5	+54 3	228 0	7·3	1	8·1	9·0	...
132	56·2364	20 7·2	+56 36	260·7	5·3	3	8·6	8·7	AB.
				60·7	37·8	3		8·6	AC.
133	56·2368	8·4	+56 56	129·4	7·8	2	8·5	9·0	...
134	63·1655	43·1	+63 6	264·3	10·4	3	8·5	9·2	...
135	56·2509	50·8	56 43	195·9	6·2	5	7·0	11·2	...
136	56·2520	56·7	56 46	340·6	5·1	1	9·2	9·3	...
137	61·2112	21 16·2	61 21	75·3	2·7	2	8·9	11·5	BC. Note.
				74·5	45·3	3		6·5	AB.
138	60·2224	17·3	60 11	265·2	8·4	2	6·5	12 8	Note.
139	52·2921	17·3	52 52		5 ±	1	9·0	11	...
140	56·2614	34·7	56 26	N.	5·	2	8·5	13·1	Too faint.
141	60·2281	38·0	60 40	186·6	1·9	1	9·5	9·6	...
142	61·2361	48·6	61 30	332·9	7·9	2	8·8	11·2	...
143	61·2363	48·8	61 30	44·9	6·5	2	8·2	13·5	Very difficult.
144	S. 800	50·3	62 3	280·8	19·8	4	7·2	12·8	Aa. Note.
				43·3	22·4	4		12·2	Bb.
				146·1	62·3	3	7·2	7·8	AB (S. 800).
145	62·2008	53·5	62 7	203·0	2·8	5	9·1	9·5	...
146	52·3140	22 8·1	+52 17	8·4	2·7	1	9·2	9·4	Poor measure.
147	54·2769	19·9	+54 16	25·1	2·0	3	8·3	10·2	AB.
				204·8	29·1	2		10·2	AC.
148	...	26·7	+61 0	286·2	3·5	2	10	10·5	Note.
149	63·2030	23 34·3	+63 39	120·9	6·0	2	8·5	8·7	...
150	64·1848	38·4	+64 23	210·0	3·1	1	9·3	11·0	...

Notes.

129. Position, October 30, 204°·2; November 7, 225°·1; probably a mistake of 20° in one or other measure.
137. The measures on both nights of BC were unsatisfactory.
138. The *comes* is too faint to measure satisfactorily.
144. S. 800. South speaks of a third star, but he probably refers to one of the more distant *comites*, the two closer ones would probably be too faint for his aperture.
148. This pair lies 22ˢ·7 f. B.D. +60°·2403 and 12″·4 S. of it.

Observations of Occultations of Stars by the Moon made at the Royal Observatory, Greenwich, in the Year 1902.

(*Communicated by the Astronomer Royal.*)

Day. 1902.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Jan. 18	Disapp. W. B. (2) II. 1358	Sheepshanks Equat.	150	Dark	5 25 13·38	A. C.
Feb. 12	,, ϵ Piscium	Great Equat.	670	,,	7 39 22·94	B.
12	,, ,,	Sheepshanks Equat.	150	,,	7 39 22·88	W. B.
12	,, ,,	Great Equat. (Corbett)	120	,,	7 39 22·94	W. H. B.
12	Reapp. ,,	Astrographic Equat.	225	Bright	8 18 11·39	W. S.
13	Disapp. Piazzi I. 223	Old Altazimuth	100	Dark	8 27 53·97	A. C.
13 (*a*)	,, ,,	Sheepshanks Equat.	150	,,	8 27 53·55	W.
13 (*b*)	,, ,,	Astro. Equat. Finder		,,	8 27 52·78	W. S.
13	,, ,,	Astrographic Equat.	225	,,	8 27 53·18	Sti.
15	,, Piazzi III. 215	Sheepshanks Equat.	150	,,	11 26 36·64	A. C.
16	,, *i* Tauri	,, ,,	150	,,	6 23 38·10	W.
16	,, ,,	Astro. Equat. Finder		,,	6 23 38·45	W. S.
16	,, ,,	Astrographic Equat.	225	,,	6 23 37·95	Sti.
March 17	,, 26 Geminorum	Sheepshanks Equat.	150	,,	9 18 50·23	A. C.
17	,, ,,	Great Equat. (Corbett)	120	,,	9 18 50·09	R.
17	,, ,,	Great Equat.	670	,,	9 18 49·99	W. B.
17	,, ,,	Astrographic Equat.	225	,,	9 18 50·17	W. S.
20 (*c*)	,, ω Leonis 1st Star	Old Altazimuth	100	,,	8 27 14·81	A. C.

Day. 1902.		Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Mar.	20 (*d*)	Disapp. ω Leonis 1st Star	Great Equat.	670	Dark	8 27 14·59	B.
	20 (*d*) (*e*)	,, ,,	Sheepshanks Equat.	150	,,	8 27 14·79	W.
	20 (*e*)	,, ,,	Astrographic Equat.	225	,,	8 27 15·35	W. S.
	20	,, ω Leonis 2nd star	Great Equat.	670	,,	8 27 16·58	B.
	20	,, ,,	Sheepshanks Equat.	150	,,	8 27 16·68	W.
	20	,, ,,	Astrographic Equat.	225	,,	8 27 16·44	W. S.
	22 (*f*) (*g*)	,, 7·5 Leonis	Sheepshanks Equat.	150	,,	11 (19 6·52)	W. S.
May	12 (*a*) (*h*)	,, 12 Cancri	Old Altazimuth	100	,,	10 37 0·62	R.
	12 (*i*)	,, ,,	Sheepshanks Equat.	150	,,	10 36 58·15	W.
	12 (*j*)	,, ,,	Astrographic Equat.	225	,,	10 36 56·24	W. S.
June	18 (*i*)	,, Piazzi XVI. 3	,, ,,	225	,,	9 36 15·46	W. S.
	18 (*i*)	,, ν Scorpii	Sheepshanks Equat.	150	,,	9 37 48·74	G. B.
	18	,, ,,	Astrographic Equat.	225	,,	9 37 48·70	W. S.
Sept.	17 (*f*) (*k*)	,, 21 Piscium	,, ,,	225	,,	11 30 (17·43)	R.
Oct.	16 (*b*) (*l*)	,, ζ¹ Piscium	Sheepshanks Equat.	150	,,	10 17 18 61	B.
	21 (*f*) (*m*)	Reapp. W. B. (2) VI. 148	Astrographic Equat.	225	,,	12 12 39 92	W. S.
Dec.	4 (*i*)	Disapp. β Capricorni	,, ,,	225	,,	7 37 44·74	W. S.

(*a*) The Moon's dark limb was easily visible.
(*b*) The observer noted "perhaps early."
(*c*) The observer suspected the fainter star for a few seconds after this.
(*d*) Instantaneous.
(*e*) The stars could just be separated before disappearance.
(*f*) These observations being uncertain have not been used.
(*g*) The signal for comparison of clocks failed to register.
(*h*) The star faded gradually.
(*i*) The star was diffused.

(*j*) This was not considered a good observation.
(*k*) The limb was boiling; the star was probably not really occulted so soon.
(*l*) The limb was almost full; the star apparently entered a deep cleft in the limb.
(*m*) Thin cloud over the Moon. The star was faint. The observer was not certain whether the star appeared from behind the Moon or the cloud.

The apertures of the telescopes used are as follows :—

	inches.		inches.
Great Equatorial	28	Great Equatorial (Corbett Telescope) ...	$6\frac{1}{2}$
Astrographic Equatorial (Guiding Telescope)	10	Old Altazimuth	4
Sheepshanks Equatorial	$6\frac{3}{4}$	Astrographic Equatorial Finder	$2\frac{1}{2}$

The initials A. C., B., R., W. B., W., G. B., W. S., W. H. B., Sti., are those of Mr. Crommelin, Mr. Bryant, Mr. Rendell, Mr. Bowyer, Mr. Witchell, Mr. Bischlager, Mr. Stevens, Mr. Brookes, and Mr. Stiles respectively.

Royal Observatory, Greenwich: 1903 *January* 9.